251.

1) $\underline{2\sin\alpha+\sqrt{2}\cos\alpha} = 2\sin\frac{\pi}{4}+\sqrt{2}\sin\frac{\pi}{4}=2\cdot\frac{\sqrt{2}}{2}+\sqrt{2}\cdot\frac{\sqrt{2}}{2}=\sqrt{2}+1$

2) $\underline{0{,}5\cos\alpha-\sqrt{3}\sin\alpha} =$

$= 0{,}5\cos\frac{\pi}{3}-\sqrt{3}\sin\frac{\pi}{3}=\frac{1}{2}\cdot\frac{1}{2}-\sqrt{3}\cdot\frac{\sqrt{3}}{2}=\frac{1}{4}-\frac{3}{2}=-\frac{5}{4}$

3) $\underline{\sin 3\alpha-\cos 2\alpha} = \sin\frac{3\pi}{6}-\cos\frac{2\pi}{6}=1-\frac{1}{2}=\frac{1}{2}$

4) $\underline{\cos\frac{\alpha}{2}+\sin\frac{\alpha}{3}} = \cos\frac{\pi}{4}+\sin\frac{\pi}{6}=\frac{\sqrt{2}}{2}+\frac{1}{2}=\frac{\sqrt{2}+1}{2}$

252.

1) $\sin x = -1$

$x = -\frac{\pi}{2} + 2\pi n$ $n \in Z$

2) $\cos x = -1$

$x = \pi + 2\pi n$ $n \in Z$

3) $\sin 3x = 0$

Тогда $3x = \pi n$, $n \in Z$

$x = \frac{\pi n}{3}$ $n \in Z$

4) $\cos 0{,}5x = 0$

Значит $0{,}5x = \frac{\pi}{2} + \pi n$, $n \in Z$

$x = \pi + 2\pi n$ $n \in Z$

5) $\cos 2x - 1 = 0$

$\cos 2x = 1$

Отсюда $2x = 2\pi n$ $n \in Z$

$x = \pi n$ $n \in Z$

6) $1 - \cos 3x = 0$

$\cos 3x = 1$

$3x = 2\pi n$, $n \in Z$

$x = \frac{2\pi n}{3}$ $n \in Z$

253.

1) $\cos 12° \approx 0{,}98$; 2) $\sin 38° \approx 0{,}62$

3) $\mathrm{tg}\,100° \approx -5{,}67$

4) $\sin 400° = \sin(360° + 40°) = \sin 40° \approx 0{,}64$

5) $\cos 2{,}7 \approx \cos 158° = \cos(180° - 22°) = -\cos 22° \approx -0{,}93$

6) $\mathrm{tg}(-13) \approx -\mathrm{tg}\,745° = -\mathrm{tg}(720° + 25°) = -\mathrm{tg}(360°\cdot 2 + 25°) =$
$= -\mathrm{tg}\,25° \approx -0{,}47$

7) $\sin\frac{\pi}{6} = 0{,}5$

8) $\cos\left(-\frac{\pi}{7}\right) \approx \cos 26° \approx 0{,}9$

254.

1) I четв.
2) II четв.
3) III четв.
4) II четв.
5) I четв.
6) II четв.

255.

1) $\sin\frac{5\pi}{4} < 0$, т.к. $\pi < \frac{5\pi}{4} < \frac{3\pi}{2}$ III четв.

2) $\sin\frac{5\pi}{6} > 0$, т.к. $\frac{\pi}{2} < \frac{5\pi}{6} < \pi$ II четв.

3) $\sin(-\frac{5\pi}{8}) < 0$, т.к. $-\pi < -\frac{5\pi}{8} < -\frac{\pi}{2}$ IV четв.

4) $\sin(-\frac{4\pi}{3}) > 0$, т.к. $-\frac{3\pi}{2} < -\frac{4\pi}{3} < -\pi$ II четв.

5) $\sin 740° > 0$, *I* четв.

6) $\sin 510° > 0$, II четв.

256.

1) $\cos\frac{2\pi}{3} < 0$, II четв. 2) $\cos\frac{7\pi}{6} < 0$, III четв.

3) $\cos(-\frac{3\pi}{4}) < 0$, III четв. 4) $\cos(-\frac{2\pi}{5}) > 0$, IV четв.

5) cos290° > 0, IV четв. 6) cos(–150°) < 0, III четв.

257.

1) $\mathrm{tg}\frac{5}{6}\pi < 0$

$\mathrm{ctg}\frac{5}{6}\pi < 0$, *II* четв.

2) $\mathrm{tg}\frac{12}{5}\pi > 0$

$\mathrm{ctg}\frac{12}{5}\pi > 0$, *II* четв.

3) $\mathrm{tg}\left(\frac{-3\pi}{5}\right) > 0$

$\mathrm{ctg}\left(\frac{-3\pi}{5}\right) > 0$, *III* четв.

4) $\mathrm{tg}\left(-\frac{5\pi}{4}\right) < 0$

$\mathrm{ctg}\left(-\frac{5\pi}{4}\right) < 0$, *II* четв.

5) tg190° > 0
ctg190° > 0, *III* четв.

6) tg283° < 0
ctg283° < 0, *I*V четв.

7) tg172° < 0
ctg172° < 0, *II* четв.

8) tg200° > 0
ctg200° > 0, *III* четв.

258.

1) если $\pi < \alpha < \frac{3\pi}{2}$, то

sinα < 0, cosα < 0, tgα > 0, ctgα > 0

2) если $\frac{3\pi}{2} < \alpha < \frac{7\pi}{4}$, то

sinα < 0, cosα > 0, tgα < 0, ctgα < 0

3) если $\frac{7\pi}{4} < \alpha < 2\pi$, то

sinα < 0, cosα > 0, tgα < 0, ctgα < 0

4) если $2\pi < \alpha < 2{,}5\pi$, то

sinα > 0, cosα > 0, tgα > 0, ctgα > 0

259.

а) sin1 > 0, cos1 > 0, tg1 > 0
б) sin3 > 0, cos3 < 0, tg3 < 0
в) sin(–3,4) > 0, cos(–3,4) < 0,
tg(–3,4) < 0
г) sin(–1,3) < 0, cos(–1,3) > 0,
tg(–1,3) < 0

260.

1) $\sin\left(\frac{\pi}{2}-\alpha\right)>0$ 2) $\cos\left(\frac{\pi}{2}+\alpha\right)<0$ 3) $\text{tg}\left(\frac{3\pi}{2}-\alpha\right)>0$

4) $\sin(\pi-\alpha)>0$ 5) $\cos(\alpha-\pi)<0$ 6) $\text{tg}(\alpha-\pi)>0$

7) $\cos\left(\alpha-\frac{\pi}{2}\right)>0$ 8) $\text{ctg}\left(\alpha-\frac{\pi}{2}\right)<0$

261.

1) если $0 < \alpha < \frac{\pi}{2}$ и $\pi < \alpha < \frac{3\pi}{2}$, то – знаки синуса и косинуса совпадают.

2) если $\frac{\pi}{2} < \alpha < \pi$ и $\frac{3\pi}{2} < \alpha < 2\pi$, то – знаки синуса и косинуса различны.

262.

1) $\sin\frac{2\pi}{3}\cdot\sin\frac{3\pi}{4}>0$

т.к. $\sin\frac{2\pi}{3}>0$ и $\sin\frac{3\pi}{4}>0$

2) $\cos\frac{2\pi}{3}\cdot\cos\frac{\pi}{6}<0$

т.к. $\cos\frac{2\pi}{3}<0$, $\cos\frac{\pi}{6}>0$

3) $\dfrac{\sin\frac{2\pi}{3}}{\cos\frac{3\pi}{4}} < 0$,

т.к. $\sin\frac{2\pi}{3} > 0$ и $\cos\frac{3\pi}{4} < 0$;

4) $\mathrm{tg}\frac{5\pi}{4} + \sin\frac{\pi}{4} > 0$,

т.к. $\mathrm{tg}\frac{5\pi}{4}$ и $\sin\frac{\pi}{4} > 0$.

263.

1) sin 0,7 > sin 4,
т.к. sin 0,7 > 0, sin4 < 0;
2) cos 1,3 > cos 2,3,
т.к. cos 1,3 > 0, cos2,3 < 0.

264.

1) $\sin(5\pi + x) = 1$;
$\sin(4\pi + \pi + x) = 1$, но
$\sin(\alpha + 2k\pi) = \sin\alpha$, где $k \in \wedge$
тогда $\sin(\pi + x) = 1$;

$\pi + x = \frac{\pi}{2} + 2\pi n$,

и $x = -\frac{\pi}{2} + 2\pi n$, $n \in \wedge$;

2) $\cos(x + 3\pi) = 0$;
$\cos(x + \pi + 2\pi) = 0$, но т.к.
$\cos(2\pi k + \alpha) = \cos\alpha$, то
$\cos(x + \pi) = 0$;

$n \in$ z $x + \pi = \frac{\pi}{2} + \pi n$, $n \in \wedge$

$x = \frac{\pi}{2} + \pi n$, $n \in \wedge$;

3) $\cos\left(\frac{5\pi}{2} + x\right) = -1$;

$\cos\left(2\pi + \frac{\pi}{2} + x\right) = -1$,

т.к. $\cos(\alpha + 2\pi k) = \cos\alpha$, то

$\cos\left(\frac{\pi}{2} + x\right) = -1$;

$\frac{\pi}{2} + x = \pi + 2\pi n$

и $x = \frac{\pi}{2} + 2\pi n$,

$n \in \wedge$;

4) $\sin\left(\frac{9}{2}\pi + x\right) = -1$;

$\sin\left(2 \cdot 2\pi + \frac{\pi}{2} + x\right) = -1$,

т.к. $\sin(2\pi k + \alpha) = \sin\alpha$, то

$\sin\left(\frac{\pi}{2} + x\right) = -1$;

$\frac{\pi}{2} + x = -\frac{\pi}{2} + 2\pi n$

и $x = \pi + 2\pi n$,

$n \in \wedge$.

265.

Т.к. $\sin\alpha + \cos\alpha < 0$, то М ∈ III четв., где $\cos\alpha < 0$, $\sin\alpha < 0$.
Т.к. $\sin\alpha - \cos\alpha > 1$, то $\sin\alpha > 0$, $\cos\alpha < 0$, значит, М ∈ II четв.

267.

1) Т.к. $\frac{3\pi}{2} < \alpha < 2\pi$, то sin α < 0, тогда

$$\sin\alpha = -\sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{25}{169}} = \sqrt{\frac{144}{169}} = \sqrt[2]{\frac{12^2}{13^2}} = -\frac{12}{13};$$

$$\text{tg}\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-12\cdot 13}{13\cdot 5} = -\frac{12}{5}.$$

2) Т.к. $\frac{\pi}{2} < \alpha < \pi$,

то cos α < 0, тогда

$$\cos\alpha = -\sqrt{1-\sin^2\alpha} = -\sqrt{1-0{,}64} = -\sqrt{0{,}36} = -0{,}6;$$

$$\text{tg}\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{0{,}8}{-0{,}6} = -\frac{4}{3}.$$

3) Т.к. $\frac{\pi}{2} < \alpha < \pi$, то sin α > 0, поэтому

$$\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \sqrt{\frac{4^2}{5^2}} = \frac{4}{5};$$

$$\text{tg}\alpha = \frac{\sin\alpha}{\cos\alpha} = -\frac{4}{5}\cdot\frac{5}{3} = -\frac{4}{3};$$

$$\text{ctg}\alpha = \frac{1}{\text{tg}\alpha} = -\frac{3}{4}.$$

4) Т.к. $\pi < \alpha < \frac{3\pi}{2}$, то cos α < 0, тогда

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\frac{4}{25}} = -\sqrt{\frac{21}{25}} = -\frac{\sqrt{21}}{5};$$

$$tg\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{2/5}{\sqrt{21}/5} = \frac{2}{\sqrt{21}};$$

$$\text{ctg}\alpha = \frac{1}{\text{tg}\alpha} = \frac{\sqrt{21}}{2}.$$

5) Т.к. $\pi < \alpha < \frac{3\pi}{2}$,

то sin α < 0 и cos α < 0;

$$\cos^2\alpha = \frac{1}{1+\text{tg}^2\alpha}; \qquad \sin\alpha = -\sqrt{1-\cos^2\alpha};$$

$\cos\alpha = -\dfrac{1}{\sqrt{1+tg^2\alpha}}$; $\quad\sin\alpha = -\sqrt{1-\dfrac{64}{289}}$;

$\cos\alpha = -\sqrt{\dfrac{64}{289}}$; $\quad\sin\alpha = -\sqrt{\dfrac{225}{289}}$;

$\cos\alpha = -\dfrac{8}{17}$; $\quad\sin\alpha = -\dfrac{15}{17}$.

6) Т.к. $\dfrac{3\pi}{2} < \alpha < 2\pi$, то $\sin\alpha < 0$, а $\cos\alpha > 0$

$\sin^2\alpha = \dfrac{1}{1+\mathrm{ctg}^2\alpha}$; $\quad\cos\alpha = \sqrt{1-\sin^2\alpha}$;

$\sin\alpha = \dfrac{-1}{\sqrt{1+\mathrm{ctg}^2\alpha}}$; $\quad\cos\alpha = \sqrt{1-\dfrac{1}{10}}$;

$\sin\alpha = -\sqrt{\dfrac{1}{10}}$; $\sin\alpha = -\dfrac{1}{\sqrt{10}}$; $\quad\cos\alpha = \dfrac{3}{\sqrt{10}}$.

268.

1) если $\begin{cases}\sin\alpha = 1\\ \cos\alpha = 1\end{cases}$,

$1 + 1 = 2 \neq 1$, нет;

2) если $\begin{cases}\sin\alpha = -\dfrac{4}{5}\\ \cos\alpha = -\dfrac{3}{5}\end{cases}$,

$\dfrac{16}{25}+\dfrac{9}{25} = 1$, да;

3) если $\begin{cases}\sin\alpha = 0\\ \cos\alpha = -1\end{cases}$,

$0 + 1 = 1$, да;

4) если $\begin{cases}\sin\alpha = \dfrac{1}{3}\\ \cos\alpha = -\dfrac{1}{2}\end{cases}$,

$\dfrac{1}{9}+\dfrac{1}{4} = \dfrac{13}{36} \neq 1$, нет.

269.

$$1+\mathrm{tg}^2\alpha=\frac{1}{\cos^2\alpha};\ 1+\mathrm{ctg}^2\alpha=\frac{1}{\sin^2\alpha};$$

1) $\begin{cases}\sin\alpha=\dfrac{1}{5}\\ tg\alpha=\dfrac{1}{\sqrt{24}}\end{cases};\qquad \begin{cases}\sin\alpha=\dfrac{1}{5}\\ ctg^2\alpha=24\end{cases};$

$$1+24=\frac{1}{\left(\frac{1}{5}\right)^2}=25.$$

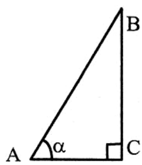
Ответ: да.

2) $\begin{cases}\cos\alpha=\dfrac{3}{4}\\ ctg\alpha=\dfrac{\sqrt{7}}{3}\end{cases};\qquad \begin{cases}\cos\alpha=\dfrac{3}{4}\\ tg^2\alpha=\dfrac{9}{7}\end{cases};$

$$1+\frac{9}{7}=\frac{1}{\left(\frac{3}{4}\right)^2},\ \frac{16}{7}\neq\frac{16}{9}.$$

Ответ: нет.

270.



Пусть: $\angle C = 90°$;

$\angle A = \alpha$;

$$\sin\alpha=\frac{2\sqrt{10}}{11};$$

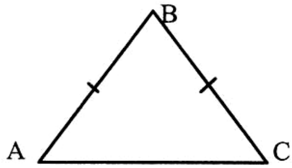
$$\cos\alpha=\sqrt{1-\sin^2\alpha};\qquad \mathrm{tg}\alpha=\frac{\sin\alpha}{\cos\alpha};$$

$$\cos\alpha=\sqrt{1-\frac{40}{121}}=\sqrt{\frac{81}{121}};\qquad \mathrm{tg}\alpha=\frac{2\sqrt{10}}{11}:\frac{9}{11};$$

$$\cos\alpha=\frac{9}{11};\qquad \mathrm{tg}\alpha=\frac{2\sqrt{10}}{9}.$$

271.

Пусть AB = BC,



tg $\angle B = 2\sqrt{2}$;

$\cos^2\alpha = \frac{1}{1+tg^2\alpha}$;

$\cos^2\alpha = \frac{1}{9}$. Т.к. $0 < \angle B < 90°$, то

$\cos\alpha = \frac{1}{3}$.

272.

$\cos^4\alpha - \sin^4\alpha = \frac{1}{8}$;

$(\cos^2\alpha - \sin^2\alpha)(\cos^2\alpha + \sin^2\alpha) = (-\cos^2\alpha - \sin^2\alpha) = \frac{1}{8}$.

Т.к. $\sin^2\alpha = 1 - \cos^2\alpha$, то $\cos^2\alpha - (1 - \cos^2\alpha) = \frac{1}{8}$;

$2\cos^2\alpha = \frac{9}{8}$ $\cos^2\alpha = \frac{9}{16}$, $\cos\alpha = \pm\frac{3}{4}$.

Ответ: $\cos\alpha = \pm\frac{3}{4}$.

273.

1) $\sin\alpha = \frac{2\sqrt{3}}{5}$;

$\cos\alpha = \pm\sqrt{1-\sin^2\alpha}$;

$\cos\alpha = \pm\sqrt{1-\frac{12}{25}}$;

$\cos\alpha = \pm\frac{\sqrt{13}}{5}$;

2) $\cos\alpha = -\frac{1}{\sqrt{5}}$;

$\sin\alpha = \pm\sqrt{1-\cos^2\alpha}$;

$\sin\alpha = \pm\sqrt{1-\frac{1}{5}}$;

$\sin\alpha = \pm\frac{2}{\sqrt{5}}$.

274.

tg α = 2, значит, ctg $\alpha = \frac{1}{2}$;

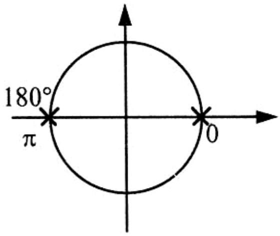
1) $\frac{\text{ctg}\alpha + \text{tg}\alpha}{\text{ctg}\alpha - \text{tg}\alpha} = \frac{1/2 + 2}{1/2 - 2} = \frac{2{,}5}{-1{,}5} = \frac{-5}{3}$;

2) $\dfrac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}=\dfrac{\dfrac{\sin\alpha}{\cos\alpha}-\dfrac{\cos\alpha}{\cos\alpha}}{\dfrac{\sin\alpha}{\cos\alpha}+\dfrac{\cos\alpha}{\cos\alpha}}=\dfrac{\mathrm{tg}\alpha-1}{\mathrm{tg}\alpha+1}=\dfrac{2-1}{2+1}=\dfrac{1}{3}$;

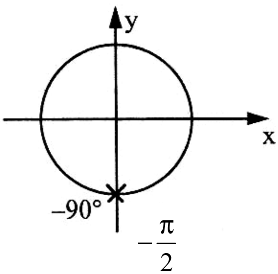
3) $\dfrac{2\sin\alpha+3\cos\alpha}{3\sin\alpha-5\cos\alpha}=\dfrac{2\mathrm{tg}\alpha+3}{3\mathrm{tg}\alpha-5}=\dfrac{4+3}{6-5}=7$;

4) $\dfrac{\sin^2\alpha+2\cos^2\alpha}{\sin^2\alpha-\cos^2\alpha}=\dfrac{\dfrac{\sin^2\alpha}{\cos^2\alpha}+2\dfrac{\cos^2\alpha}{\cos^2\alpha}}{\dfrac{\sin^2\alpha}{\cos^2\alpha}-\dfrac{\cos^2\alpha}{\cos^2\alpha}}=\dfrac{\mathrm{tg}^2\alpha+2}{\mathrm{tg}^2\alpha-1}=\dfrac{4+2}{4-1}=2$.

276.



1) $2\sin x+\sin^2 x+\cos^2 x=1$,
т.к. $\sin^2 x+\cos^2 x=1$, то
$2\sin x+1=1$,
$2\sin x=0$.
Тогда $\sin x=0$
и $x=k\pi$, $k\in\wedge$;



2) $\sin^2 x-2=\sin x-\cos^2 x$;
$\sin^2 x+\cos^2 x-2=\sin x$,
т.к. $\sin^2 x+\cos^2 x=1$, то
$\sin x=-1$,
значит, $x=-\dfrac{\pi}{2}+2\pi n,\ n\in\wedge$;

3) $3\cos^2 x-1=\cos x-2\sin^2 x$;
$3\cos^2 x+2\sin^2 x-1=\cos x$;
$\cos^2 x+2-1=\cos x$;
$\cos^2 x-\cos x+1=0$.
Пусть $t=\cos x$. Тогда
$t^2-t+1=0$. Решим уравнение
$D=1-4<0$. Решения нет.

4) $3-\cos x=3\cos^2 x+3\sin^2 x$.
Т.к. $\sin^2 x+\cos^2 x=1$, то
$3-\cos x=3$;
$\cos x=0$;
$x=\dfrac{\pi}{2}+\pi n;\ n\in\wedge$.

277.

1) Т.к. $1 - \cos^2\alpha = \sin^2\alpha$, то $(1-\cos\alpha)(1+\cos\alpha)=\sin^2\alpha$.

2) Т.к. $\sin^2\alpha+\cos^2\alpha=1$, то $2-\sin^2\alpha-\cos^2\alpha=1$.

3) Т.к. $tg^2\alpha = \dfrac{\sin^2\alpha}{\cos^2\alpha}$ и $\cos^2\alpha = 1-\sin^2\alpha$, то $\dfrac{\sin^2\alpha}{1-\sin^2\alpha} = \mathrm{tg}^2\alpha$.

4) Т.к. $ctg^2\alpha = \dfrac{\cos^2\alpha}{\sin^2\alpha}$ и $\sin^2\alpha = 1-\cos^2\alpha$, то $\dfrac{\cos^2\alpha}{1-\cos^2\alpha} = \mathrm{ctg}^2\alpha$.

5) Т.к. $\cos^2\alpha + \sin^2\alpha = 1$ и $\cos^2\alpha=\dfrac{1}{1+tg^2\alpha}$, то $\dfrac{1}{1+\mathrm{tg}^2\alpha}+\sin^2\alpha=1$.

6) Т.к. $\sin^2\alpha + \cos^2\alpha = 1$ и $\sin^2\alpha = \dfrac{1}{1+ctg^2\alpha}$, то $\dfrac{1}{1+\mathrm{ctg}^2\alpha}+\cos^2\alpha=1$.

278.

$\cos\alpha \cdot \mathrm{tg}\alpha - 2\sin\alpha = \sin\alpha - 2\sin\alpha = -\sin\alpha$;

$\cos\alpha - \sin\alpha \cdot \mathrm{ctg}\alpha = \cos\alpha - \cos\alpha = 0$;

$$\frac{\sin^2\alpha}{1+\cos\alpha} = \frac{1\quad\cos^2\alpha}{1+\cos\alpha} = \frac{(1+\cos\alpha)(1-\cos\alpha)}{1+\cos\alpha} = 1-\cos\alpha;$$

$$\frac{\cos^2\alpha}{1-\sin\alpha} = \frac{1-\sin^2\alpha}{1-\sin\alpha} = \frac{(1+\sin\alpha)(1-\sin\alpha)}{1-\sin\alpha} = 1+\sin\alpha.$$

279.

1) $\dfrac{\sin^2\alpha-1}{1-\cos^2\alpha} = \dfrac{-\cos^2\alpha}{\sin^2\alpha} = -\mathrm{ctg}^2\alpha$; $\mathrm{ctg}\dfrac{\pi}{4}=1$; $-\mathrm{ctg}^2\dfrac{\pi}{4} = -1$;

2) $\dfrac{1}{\cos^2\alpha}-1 = \mathrm{tg}^2\alpha$; $\mathrm{tg}\dfrac{\pi}{3} = \sqrt{3}$; $\mathrm{ctg}^2\dfrac{\pi}{3} = 3$;

3) $\cos^2\alpha+\mathrm{ctg}^2\alpha+\sin^2\alpha = 1+\mathrm{ctg}^2\alpha = \dfrac{1}{\sin^2\alpha}$,

$\sin\dfrac{\pi}{6} = \dfrac{1}{2}$, $\dfrac{1}{\sin^2\frac{\pi}{6}} = 4$;

4) $\cos^2\alpha+\mathrm{tg}^2\alpha+\sin^2\alpha = 1+\mathrm{tg}^2\alpha = \dfrac{1}{\cos^2\alpha}$,

$\cos\dfrac{\pi}{3} = \dfrac{1}{2}$, $\dfrac{1}{\cos^2\frac{\pi}{3}} = 4$.

280.

1) $\left(1-\sin^2\alpha\right)\left(1-\mathrm{tg}^2\alpha\right)=1$.

Тогда $\left(1-\sin^2\alpha\right)\cdot\frac{1}{\cos^2\alpha}=1$;

$\cos^2\alpha\cdot\frac{1}{\cos^2\alpha}=1$, 1 = 1.

Получим тождество.

2) $\sin^2\left(1+\mathrm{ctg}^2\alpha\right)-\cos^2\alpha=\sin^2\alpha$.

Значит $\sin^2\alpha\cdot\frac{1}{\sin^2\alpha}-\cos^2\alpha=\sin^2\alpha$;

$1-\cos^2\alpha=\sin^2\alpha$.

Тождество $\sin^2\alpha=\sin^2\alpha$.

281.

1) $\left(1+\mathrm{tg}^2\alpha\right)\cdot\cos^2\alpha=\frac{1}{\cos^2\alpha}\cdot\cos^2\alpha=1;$

2) $\sin^2\alpha\left(1+\mathrm{ctg}^2\alpha\right)=\sin^2\alpha\cdot\frac{1}{\sin^2\alpha}=1;$

3) $\left(1+\mathrm{tg}^2\alpha+\frac{1}{\sin^2\alpha}\right)\sin^2\alpha\cdot\cos^2\alpha=\frac{\sin^2\alpha+\cos^2\alpha}{\sin^2\alpha\cdot\cos^2\alpha}\cdot\sin^2\alpha\cdot\cos^2\alpha=1$;

4) $\frac{1+\mathrm{tg}^2\alpha}{1+\mathrm{ctg}^2\alpha}-\mathrm{tg}^2\alpha=\frac{1/\cos^2\alpha}{1/\sin^2\alpha}-\mathrm{tg}^2\alpha=\frac{\sin^2\alpha}{\cos^2\alpha}-\mathrm{tg}^2\alpha=0$.

282.

1) $(1-\cos2\alpha)(1+\cos2\alpha)=\sin^2 2\alpha;$

$1-\cos^2 2\alpha=\sin^2 2\alpha;$

$\sin^2 2\alpha=\sin^2 2\alpha$. Верное тождество.

2) $\frac{\sin\alpha-1}{\cos^2\alpha}=\frac{-1}{1+\sin\alpha}$;

$\frac{\sin\alpha-1}{1-\sin^2\alpha}=\frac{-1}{1+\sin\alpha}$;

$\frac{\sin\alpha-1}{(1-\sin\alpha)(1+\sin\alpha)}=-\frac{1}{1+\sin\alpha}$;

$\frac{1}{-(1+\sin\alpha)}=-\frac{1}{1+\sin\alpha}$. Верно.

3) $\cos^4\alpha - \sin^4\alpha = \cos^2\alpha - \sin^2\alpha$;
$(\cos^2\alpha + \sin^2\alpha)(\cos^2\alpha - \sin^2\alpha) = \cos^2\alpha - \sin^2\alpha$;
$\cos^2\alpha - \sin^2\alpha = \cos^2\alpha - \sin^2\alpha$. Верное тождество.
4) $(\sin^2\alpha - \cos^2\alpha)^2 + 2\sin^2\alpha \cdot \cos^2\alpha = \sin^4\alpha + \cos^4\alpha$;
$\sin^4\alpha - 2\sin^2\alpha \cdot \cos^2\alpha + \cos^4\alpha + 2\sin^2\alpha \cdot \cos^2\alpha = \sin^4\alpha + \cos^4\alpha$;
$\sin^4\alpha + \cos^4\alpha = \sin^4\alpha + \cos^4\alpha$. Верное тождество.
5) $\frac{\sin\alpha}{1+\cos\alpha} + \frac{1+\cos\alpha}{\sin\alpha} = \frac{2}{\sin\alpha}$;

$\frac{\sin^2\alpha + (1+\cos\alpha)^2}{(1+\cos\alpha)\sin\alpha} = \frac{2}{\sin\alpha}$;

$\frac{\sin^2\alpha + 1 + 2\cos\alpha + \cos^2\alpha}{(1+\cos\alpha)\sin\alpha} = \frac{2}{\sin\alpha}$;

$\frac{2(1+\cos\alpha)}{(1+\cos\alpha)\sin\alpha} = \frac{2}{\sin\alpha}$; $\frac{2}{\sin\alpha} = \frac{2}{\sin\alpha}$. Верное тождество.

6) $\frac{\sin\alpha}{1-\cos\alpha} = \frac{1+\cos\alpha}{\sin\alpha}$;

$\frac{\sin\alpha(1+\cos\alpha)}{(1+\cos\alpha)(1-\cos\alpha)} = \frac{1+\cos\alpha}{\sin\alpha}$;

$\frac{\sin\alpha(1+\cos\alpha)}{1-\cos^2\alpha} = \frac{1+\cos\alpha}{\sin\alpha}$;

$\frac{\sin\alpha(1+\cos\alpha)}{\sin^2\alpha} = \frac{1+\cos\alpha}{\sin\alpha}$;

$\frac{1+\cos\alpha}{\sin\alpha} = \frac{1+\cos\alpha}{\sin\alpha}$. Верное тождество.

7) $\frac{1}{1+\mathrm{tg}^2\alpha} + \frac{1}{1+\mathrm{ctg}^2\alpha} = 1$;

$\cos^2\alpha + \sin^2\alpha = 1$; $1 = 1$, ч.т.д.
8) $\mathrm{tg}^2\alpha - \sin^2\alpha = \mathrm{tg}^2\alpha \cdot \sin^2\alpha$;

$\frac{\sin^2\alpha}{\cos^2\alpha} - \sin^2\alpha = \mathrm{tg}^2\alpha \cdot \sin^2\alpha$;

$\frac{\sin^2\alpha - \sin^2\alpha \cdot \cos^2\alpha}{\cos^2\alpha} = \mathrm{tg}^2\alpha \cdot \sin^2\alpha$;

$\frac{\sin^2\alpha\left(1-\cos^2\alpha\right)}{\cos^2\alpha} = \mathrm{tg}^2\alpha \cdot \sin^2\alpha$;

$\mathrm{tg}^2\alpha \cdot \sin^2\alpha = \mathrm{tg}^2\alpha \cdot \sin^2\alpha$, ч.т.д.

283.

1) $$\frac{(\sin\alpha+\cos\alpha)^2}{\sin^2\alpha}-\left(1+ctg^2\alpha\right)=\frac{1-2\sin\alpha\cos\alpha}{\sin^2\alpha}-\frac{1}{\sin^2\alpha}=$$

$$=\frac{2\sin\alpha\cos\alpha}{\sin^2\alpha}=2\mathrm{ctg}\alpha\ ;$$

$$\mathrm{ctg}\ \frac{\pi}{3}=\frac{1}{\sqrt{3}}\ ;$$

$$2\mathrm{ctg}\frac{\pi}{3}=\frac{2}{\sqrt{3}}=\frac{2\sqrt{3}}{3}\ ;$$

2) $$(1+tg^2\alpha)-\frac{(\sin\alpha-\cos\alpha)^2}{\cos^2\alpha}=\frac{1}{\cos^2\alpha}-\frac{1+2\sin\alpha\cos\alpha}{\cos^2\alpha}=$$

$$=\frac{2\sin\alpha\cos\alpha}{\cos^2\alpha}=2tg\alpha\ ;$$

$$\mathrm{tg}\frac{\pi}{6}=\frac{1}{\sqrt{3}}\ ;$$

$$2\mathrm{tg}\frac{\pi}{6}=\frac{2}{\sqrt{3}}=\frac{2\sqrt{3}}{3}\ .$$

284.

$\sin\alpha - \cos\alpha = 0{,}6$.

Возведем в квадрат

$(\sin\alpha - \cos\alpha)^2 = 0{,}36$;

$\sin^2\alpha - 2\sin\alpha\cos\alpha + \cos^2\alpha = 0{,}36$.

Т.к. $\sin^2\alpha + \cos^2\alpha = 1$, то

$1 - 2\sin\alpha\cos\alpha = 0{,}36$;

$2\sin\alpha\cos\alpha = 1 - 0{,}36 = 0{,}64$;

$\sin\alpha\cos\alpha = 0{,}32$.

285.

$\cos^3\alpha - \sin^3\alpha = (\cos\alpha - \sin\alpha)(\cos^2\alpha + \cos\alpha\cdot \sin\alpha + \sin^2\alpha)$;

$\cos^3\alpha - \sin^3\alpha = 0{,}2 \cdot (1 + \cos\alpha\cdot \sin\alpha)$;

т.к. $\cos\alpha–\sin\alpha = 0{,}2$. Возведем в квадрат

$(\cos\alpha - \sin\alpha)^2=0{,}04$;

$\cos^2\alpha - 2\cos\alpha\sin\alpha+\sin^2\alpha=0{,}04$;

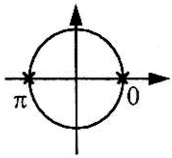
$1–2 \cos\alpha\sin\alpha= 0{,}04$;

$\cos\alpha\sin\alpha=0{,}48$, то

$\cos^3\alpha - \sin^3\alpha = 0{,}2 \cdot (1 + 0{,}48) = 0{,}2 \cdot 1{,}48 = 0{,}296$.

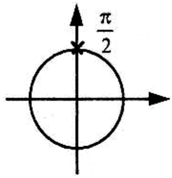
286.

1) $3\cos^2 x - 2\sin x = 3 - 3\sin^2 x$;
$3\cos^2 x + 3\sin^2 x - 3 - 2\sin x = 0$;
$2\sin x = 0$;
$\sin x = 0$.
Тогда $x = \pi n$, $n \in \wedge$.



2) $\cos^2 x - \sin^2 x = 2\sin x - 1 - 2\sin^2 x$;
$\cos^2 x - \sin^2 x + 1 + 2\sin^2 x = 2\sin x$;
$2 = 2\sin x$;
$\sin x = 1$.

Значит $x = \dfrac{\pi}{2} + 2\pi n$, $n \in \wedge$.



287.

$$1)\ \cos\left(-\frac{\pi}{6}\right)\sin\left(-\frac{\pi}{3}\right)+tg\left(-\frac{\pi}{4}\right)=-\cos\frac{\pi}{6}\cdot\sin\frac{\pi}{3}-tg\frac{\pi}{4}=$$

$$=-\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}-1=-\frac{3}{4}-1=-1\frac{3}{4};$$

$$2)\ \frac{1+\text{tg}^2(\ \ 30°)}{1+\text{ctg}^2(\ \ 30°)}=\frac{1+\text{tg}^2 30°}{1+\text{ctg}^2 30°}=\frac{1+\frac{1}{3}}{1+3}=\frac{3+1}{3\cdot 4}=\frac{4}{3\cdot 4}=\frac{1}{3};$$

$$3)\ 2\sin\left(-\frac{\pi}{6}\right)\cos\left(-\frac{\pi}{6}\right)+tg\left(-\frac{\pi}{3}\right)+\sin^2\left(-\frac{\pi}{4}\right)=$$

$$=-2\sin\frac{\pi}{6}\cos\frac{\pi}{6}-tg\frac{\pi}{3}+\sin^2\frac{\pi}{4}=-2\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{2}-\sqrt{3}+\left(\frac{\sqrt{2}}{2}\right)^2=$$

$$=-\frac{\sqrt{3}}{2}-\frac{2\sqrt{3}}{2}+\frac{1}{2}=\frac{1-3\sqrt{3}}{2};$$

$$4)\ \cos(-\pi)+ctg\left(-\frac{\pi}{2}\right)-\sin\left(\frac{3}{2}\pi\right)+ctg\left(-\frac{\pi}{4}\right)=$$

$$=\cos\pi-ctg\frac{\pi}{2}+\sin\frac{3}{2}\pi-ctg\frac{\pi}{4}=-1+0+(-1)-1=-3.$$

288.

$\text{tg}(-\alpha)\cdot\cos\alpha+\sin\alpha=-\sin\alpha+\sin\alpha=0$;
$\cos\alpha-\text{ctg}\alpha(-\sin\alpha)=\cos\alpha+\cos\alpha=2\cos\alpha$;

$$\frac{\cos(\ \alpha)+\sin(\ \alpha)}{\cos^2\alpha-\sin^2\alpha}=\frac{\cos\alpha-\sin\alpha}{(\cos\alpha+\sin\alpha)(\cos\alpha-\sin\alpha)}=\frac{1}{\cos\alpha+\sin\alpha};$$

$\mathrm{tg}(-\alpha)\cdot\mathrm{ctg}(-\alpha)+\cos^2(-\alpha)+\sin^2\alpha=1+1=2.$

289.

$$\frac{\cos^2\alpha-\sin^2\alpha}{\cos\alpha+\sin(-\alpha)}+\mathrm{tg}(-\alpha)\cos(-\alpha)=$$

$$=\frac{(\cos\alpha+\sin\alpha)(\cos\alpha-\sin\alpha)}{\cos\alpha-\sin\alpha}\quad\sin\alpha=$$

$=\cos\alpha+\sin\alpha-\sin\alpha=\cos\alpha.$

290.

1) $$\frac{3-\sin\left(-\frac{\pi}{3}\right)-\cos^2\left(-\frac{\pi}{3}\right)}{2\cos\left(-\frac{\pi}{4}\right)}=\frac{3+\sin\frac{\pi}{3}-\cos^2\frac{\pi}{3}}{2\cos\frac{\pi}{4}}=\frac{3+\frac{\sqrt{3}}{2}-\frac{1}{4}}{2\cdot\frac{\sqrt{2}}{2}}=\frac{11+2\sqrt{3}}{4\cdot\sqrt{2}};$$

2) $$2\sin\left(-\frac{\pi}{6}\right)-3\mathrm{ctg}\left(-\frac{\pi}{4}\right)+7{,}5\mathrm{tg}(-\pi)+\frac{1}{8}\cos\left(-\frac{3}{2}\pi\right)=$$

$$=2\cdot\left(-\frac{1}{2}\right)-3\cdot(-1)+7{,}5\cdot0+\frac{1}{8}\cdot0=-1+3=2.$$

291.

1) $$\frac{\sin^3(-\alpha)+\cos^3(-\alpha)}{1-\sin(-\alpha)\cos(-\alpha)}=$$

$$=\frac{(\cos\alpha-\sin\alpha)(\cos^2\alpha+\cos\alpha\sin\alpha+\sin^2\alpha)}{1+\sin\alpha\cos\alpha}=$$

$$=\frac{(\cos\alpha-\sin\alpha)(1+\cos\alpha\sin\alpha)}{(1+\cos\alpha\sin\alpha)}=\cos\alpha-\sin\alpha;$$

2) $$\frac{1-(\sin\alpha+\cos(-\alpha))^2}{-\sin(-\alpha)}=\frac{1-(1+2\sin\alpha\cos\alpha)}{\sin\alpha}=\frac{-2\sin\alpha\cos\alpha}{\sin\alpha}=-2\cos\alpha.$$

292.

1) $\sin(-x)=1$;
$\sin x=-1$.

Тогда $x=-\frac{\pi}{2}+2\pi n,\ n\in\wedge$.

2) $\cos(-2x)=0$;
$\cos 2x=0$;

$2x=\frac{\pi}{2}+\pi n$.

Значит, $x=\frac{\pi}{4}+\frac{\pi n}{2},\ n\in\wedge$.

3) $\cos(-2x) = 1$;
$\cos 2x = 1$;
$2x = 2\pi n$;
и $x = \pi n$, $n \in \wedge$.

4) $\sin(-2x) = 0$;
$2x = 2\pi n$.
Поэтому $x = \dfrac{\pi n}{2}$, $n \in \wedge$.

5) $\sin(-x) = \sin\dfrac{3}{2}\pi$;
$-\sin x = -1$; $\sin x = 1$.
Получим $x = \dfrac{\pi}{2} + 2\pi n$, $n \in \wedge$.

6) $\cos(-x) = \cos\pi$;
$\cos x = -1$.
Тогда $x = \pi + 2\pi n$, $n \in \wedge$.

293.

1) $\cos 135° = \cos(90° + 45°) = \cos 90° \cos 45° - \sin 90° \cdot \sin 45° =$
$= 0 \cdot \dfrac{\sqrt{2}}{2} - 1 \cdot \dfrac{\sqrt{2}}{2} = -\dfrac{\sqrt{2}}{2}$;

2) $\cos 120° = \cos(90° + 30°) = \cos 90° \cos 30° - \sin 90° \sin 30° =$
$= 0 \cdot \dfrac{\sqrt{3}}{2} - 1 \cdot \dfrac{1}{2} = -\dfrac{1}{2}$;

3) $\cos 150° = \cos(90° + 60°) = \cos 90° \cos 60° - \sin 90° \sin 60° =$
$= 0 \cdot \dfrac{1}{2} - 1 \cdot \dfrac{\sqrt{3}}{2} = -\dfrac{\sqrt{3}}{2}$;

4) $\cos 240° = \cos(180° + 60°) = \cos 180° \cos 60° - \sin 180° \sin 60° =$
$= -1 \cdot \dfrac{1}{2} - 0 \cdot \dfrac{\sqrt{3}}{2} = -\dfrac{1}{2}$.

294.

1) $\cos 57°30' \cdot \cos 27°30' + \sin 57°30' \cdot \sin 27°30' =$
$= \cos(57°30' - 27°30') = \cos 30° = \dfrac{\sqrt{3}}{2}$;

2) $\cos 19°30' \cdot \cos 25°30' - \sin 19°30' \cdot \sin 25°30' =$
$= \cos(19°30' - 25°30') = \cos 45° = \dfrac{\sqrt{2}}{2}$;

3) $\cos\dfrac{7\pi}{9} \cdot \cos\dfrac{11\pi}{9} - \sin\dfrac{7\pi}{9} \cdot \sin\dfrac{11\pi}{9} = \cos\left(\dfrac{7\pi}{9} + \dfrac{11\pi}{9}\right) = \cos 2\pi = 1$;

4) $\cos\dfrac{8\pi}{7} \cdot \cos\dfrac{\pi}{7} + \sin\dfrac{8\pi}{7} \cdot \sin\dfrac{\pi}{7} = \cos\left(\dfrac{8\pi}{7} + \dfrac{\pi}{7}\right) = \cos\pi = -1$.

295.

1) Т.к. $0 < \alpha < \dfrac{\pi}{2}$, то

$\cos\alpha > 0$, тогда

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{\frac{2}{3}};$$

$$\cos\left(\frac{\pi}{3}+\alpha\right) = \cos\frac{\pi}{3}\cdot\cos\alpha - \sin\frac{\pi}{3}\cdot\sin\alpha =$$

$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{\sqrt{3}} - \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{3}} = \frac{\sqrt{2}-\sqrt{3}}{2\sqrt{3}}.$$

2) Т.к. $\dfrac{\pi}{2} < \alpha < \pi$, то

$\sin\alpha > 0$, тогда

$$\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{1}{9}} = \frac{2\sqrt{2}}{\sqrt{3}};$$

$$\cos\left(\alpha-\frac{\pi}{4}\right) = \cos\alpha\cdot\cos\frac{\pi}{4} + \sin\alpha\cdot\sin\frac{\pi}{4} = -\frac{1}{3}\cdot\frac{\sqrt{2}}{2} + \frac{2\sqrt{2}}{3}\cdot\frac{\sqrt{2}}{2} = \frac{4-\sqrt{2}}{6}.$$

296.

1) $\cos3\alpha\cdot\cos\alpha - \sin\alpha\cdot\sin3\alpha = \cos(3\alpha+\alpha) = \cos4\alpha$;

2) $\cos5\beta\cdot\cos2\beta + \sin5\beta\cdot\sin2\beta = \cos(5\beta-2\beta) = \cos3\beta$;

3) $$\cos\left(\frac{\pi}{7}+\alpha\right)\ \cos\left(\frac{5\pi}{14}-\alpha\right)\ \sin\left(\frac{\pi}{7}+\alpha\right)\ \sin\left(\frac{5\pi}{14}-\alpha\right) =$$

$$= \cos\left(\frac{\pi}{7}+\alpha+\frac{5\pi}{14}-\alpha\right) = \cos\frac{\pi}{2} = 0;$$

4) $$\cos\left(\frac{7\pi}{5}+\alpha\right)\cdot\cos\left(\frac{2\pi}{5}+\alpha\right) + \sin\left(\frac{7\pi}{5}+\alpha\right)\cdot\sin\left(\frac{2\pi}{5}+\alpha\right) =$$

$$= \cos\left(\frac{7\pi}{5}+\alpha-\frac{2\pi}{5}-\alpha\right) = \cos\pi = -1.$$

297.

1) $$\cos(\alpha+\beta) + \cos\left(\frac{\pi}{2}-\alpha\right)\cos\left(\frac{\pi}{2}-\beta\right) = \cos\alpha\cdot\cos\beta -$$

$$-\sin\alpha\cdot\sin\beta + \sin\alpha\cdot\sin\beta = \cos\alpha\cdot\cos\beta;$$

2) $\sin\left(\frac{\pi}{2}-\alpha\right)\sin\left(\frac{\pi}{2}-\beta\right)-\cos(\alpha-\beta)=\left(\sin\frac{\pi}{2}\cdot\cos\alpha-\cos\frac{\pi}{2}\cdot\sin\alpha\right)$x

x $\left(\sin\frac{\pi}{2}\cdot\cos\beta-\cos\frac{\pi}{2}\cdot\sin\beta\right)-(\cos\alpha\cdot\cos\beta+\sin\alpha\cdot\sin\beta)=$

$=\cos\alpha\cdot\cos\beta-\cos\alpha\cdot\cos\beta-\sin\alpha\cdot\sin\beta=-\sin\alpha\cdot\sin\beta$.

298.

1) sin73° · cos17° + cos73° · sin17° = sin(73° + 17°)=sin90°=1;

2) sin73° · cos13° – cos73° · sin13° = sin(73° – 13°)=sin60°=$\frac{\sqrt{3}}{2}$;

3) $\sin\frac{5\pi}{12}\cdot\cos\frac{\pi}{12}+\sin\frac{\pi}{12}\cdot\cos\frac{5\pi}{12}=\sin\left(\frac{5\pi}{12}+\frac{\pi}{12}\right)=\sin\frac{\pi}{2}=1$;

4) $\sin\frac{7\pi}{12}\cdot\cos\frac{\pi}{12}-\sin\frac{\pi}{12}\cdot\cos\frac{7\pi}{12}=\sin\left(\frac{7\pi}{12}-\frac{\pi}{12}\right)=\sin\frac{\pi}{2}=1$.

299.

1) Т.к. $\pi<\alpha<\frac{3\pi}{2}$, то

$\sin\alpha<0$, тогда

$\sin\alpha=-\sqrt{1-\cos^2\alpha}=-\sqrt{1-\frac{9}{25}}=-\frac{4}{5}$;

$\sin\left(\alpha+\frac{\pi}{6}\right)=\sin\alpha\cdot\cos\frac{\pi}{6}+\cos\alpha\cdot\sin\frac{\pi}{6}=-\frac{4}{5}\cdot\frac{\sqrt{3}}{2}-\frac{3}{5}\cdot\frac{1}{2}=$

$=\frac{-4\sqrt{3}-3}{10}=-\frac{4\sqrt{3}+3}{10}$.

2) Т.к. $\frac{\pi}{2}<\alpha<\pi$, то

$\cos\alpha<0$, тогда

$\cos\alpha=-\sqrt{1-\sin^2\alpha}=-\sqrt{1-\frac{2}{9}}=\frac{\sqrt{7}}{3}$;

$\sin\left(\frac{\pi}{4}-\alpha\right)=\sin\frac{\pi}{4}\cdot\cos\alpha-\cos\frac{\pi}{4}\cdot\sin\alpha=\frac{\sqrt{2}}{2}\cdot\left(\frac{\sqrt{7}}{3}\right)-\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{3}=$

$=\frac{-\sqrt{14}-2}{6}=-\frac{\sqrt{14}+2}{6}$.

300.

1) $\sin(\alpha + \beta) + \sin(-\alpha)\cos(-\beta) = \sin\alpha\cdot\cos\beta +$
$+ \cos\alpha\cdot\sin\beta - \sin\alpha\cdot\cos\beta = \cos\alpha \cdot \sin\beta$;

2) $\cos(-\alpha)\sin(-\beta) - \sin(\alpha - \beta) =$
$= -\cos\alpha\cdot \sin\beta - (\sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta) = -\cos\alpha\cdot\sin\beta - \sin\alpha\cdot\cos\beta +$
$+ \cos\alpha\cdot\sin\beta = -\sin\alpha\cdot\cos\beta$;

3)

3) $\cos\left(\frac{\pi}{2}-\alpha\right)\sin\left(\frac{\pi}{2}-\beta\right)-\sin(\alpha-\beta)=\left(\cos\frac{\pi}{2}\cos\alpha+\sin\frac{\pi}{2}\sin\alpha\right)\times$
$\times\left(\sin\frac{\pi}{2}\cos\beta-\cos\frac{\pi}{2}\sin\beta\right)-\sin\alpha\cos\beta+\cos\alpha\sin\beta=\sin\alpha\cos\beta-$
$-\sin\alpha\cos\beta+\cos\alpha\sin\beta=\cos\alpha\sin\beta$;

4) $\sin(\alpha+\beta)+\sin\left(\frac{\pi}{2}-\alpha\right)\sin(-\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta-$
$-\cos\alpha\sin\beta=\sin\alpha\cos\beta$.

301.

Т.к. $\frac{3\pi}{2}<\alpha<2\pi$, то

$\cos\alpha > 0$,

тогда

$\cos\alpha=\sqrt{1-\sin^2\alpha}=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$.

Т.к. $0<\beta<\frac{\pi}{2}$, то

$\cos\beta > 0$,

тогда

$\cos\beta=\sqrt{1-\sin^2\alpha}=\sqrt{1-\frac{64}{289}}=\frac{15}{17}$;

$\cos(\alpha + \beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta =$
$=\frac{4}{5}\cdot\frac{15}{17}-\left(-\frac{3}{5}\right)\cdot\frac{8}{17}=\frac{60}{85}+\frac{24}{85}=\frac{84}{85}$;

$\cos(\alpha - \beta) = \frac{4}{5}\cdot\frac{15}{17}+\left(-\frac{3}{5}\right)\cdot\frac{8}{17}=\frac{60}{85}-\frac{24}{85}=\frac{36}{85}$.

302.

Т.к. $\frac{\pi}{2} < \alpha < \pi$, то $\sin\alpha > 0$;

$\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-0{,}64} = \sqrt{0{,}36} = 0{,}6$.

Т.к. $\pi < \beta < \frac{3\pi}{2}$,

то $\cos\beta < 0$;

$\cos\beta = -\sqrt{1-\sin^2\alpha} = -\sqrt{1-\frac{144}{169}} = -\frac{5}{13}$;

$\sin(\alpha - \beta) = \sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta =$

$= 0{,}6\cdot\left(-\frac{5}{13}\right) - (-0{,}8)\cdot\left(-\frac{12}{13}\right) = \frac{-15}{65} - \frac{48}{65} = -\frac{63}{65}.$

303.

1) $\cos\left(\frac{2}{3}\pi-\alpha\right)+\cos\left(\alpha+\frac{\pi}{3}\right) = \cos\frac{2\pi}{3}\cdot\cos\alpha+\sin\frac{2\pi}{3}\cdot\sin\alpha +$

$+\cos\frac{\pi}{3}\cdot\cos\alpha-\sin\frac{\pi}{3}\cdot\sin\alpha = -\frac{1}{2}\cos\alpha+\frac{\sqrt{3}}{2}\sin\alpha+\frac{1}{2}\cos\alpha-\frac{\sqrt{3}}{2}\sin\alpha = 0$;

2) $\sin\left(\alpha+\frac{2}{3}\pi\right)-\sin\left(\frac{\pi}{3}-\alpha\right) = \sin\alpha\cdot\cos\frac{2\pi}{3}+\cos\alpha\cdot\sin\frac{2\pi}{3} -$

$-\sin\frac{\pi}{3}\cdot\cos\alpha+\cos\frac{\pi}{3}\cdot\sin\alpha = -\frac{1}{2}\sin\alpha+\frac{\sqrt{3}}{2}\cos\alpha-\frac{\sqrt{3}}{2}\cos\alpha+\frac{1}{2}\sin\alpha = 0$;

3) $\frac{2\cos\alpha\sin\beta+\sin(\alpha-\beta)}{2\cos\alpha\cos\beta-\cos(\alpha-\beta)} = \frac{2\cos\alpha\sin\beta+\sin\alpha\cos\beta-\cos\alpha\sin\beta}{2\cos\alpha\cos\beta-\cos\alpha\cos\beta-\sin\alpha\sin\beta} =$

$= \frac{\cos\alpha\sin\beta+\sin\alpha\cos\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta} = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = tg(\alpha+\beta)$;

4) $\frac{\cos\alpha\cos\beta-\cos(\alpha+\beta)}{\cos(\alpha-\beta)-\sin\alpha\sin\beta} = \frac{\cos\alpha\cos\beta-\cos\alpha\cos\beta+\sin\alpha\sin\beta}{\cos\alpha\cos\beta+\sin\alpha\sin\beta-\sin\alpha\sin\beta} =$

$= \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta} = tg\,\alpha\cdot tg\beta$.

304.

1) $\sin(\alpha - \beta)\cdot\cos(\alpha + \beta) = (\sin\alpha\cos\beta - \cos\alpha\,\sin\beta)(\sin\alpha\cos\beta + \cos\alpha\,\sin\beta) = \sin^2\alpha\,\cos^2\beta - \cos^2\alpha\,\sin^2\beta = \sin^2\alpha(1 - \sin^2\beta) - (1 - \sin^2\alpha)\sin^2\beta = \sin^2\alpha - \sin^2\alpha\cdot\sin^2\beta - \sin^2\beta + \sin^2\alpha\cdot\sin^2\beta = \sin^2\alpha - \sin^2\beta$;

2) $\sin(\alpha - \beta)\cdot\cos(\alpha + \beta) = (\cos\alpha\cos\beta - \sin\alpha \sin\beta)(\cos\alpha\cos\beta +$
$+ \sin\alpha \sin\beta) = \cos^2\alpha \cos^2\beta - \sin^2\alpha \sin^2\beta = \cos^2\alpha(1 - \sin^2\beta) -$
$- (1 - \cos^2\alpha) \sin^2\beta = \cos^2\alpha - \cos^2\alpha\cdot \sin^2\beta - \sin^2\beta + \cos^2\alpha \cdot \sin^2\beta =$
$= \cos^2\alpha - \sin^2\beta;$

3) $$\frac{\sqrt{2}\cos\alpha - 2\cos\left(\frac{\pi}{4} - \alpha\right)}{2\sin\left(\frac{\pi}{6} + \alpha\right) - \sqrt{3}\sin\alpha} = \frac{\sqrt{2}\cos\alpha - 2\left(\frac{\sqrt{2}}{2}\cos\alpha + \frac{\sqrt{2}}{2}\sin\alpha\right)}{2\left(\frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2}\sin\alpha\right) - \sqrt{3}\sin\alpha} =$$

$$= \frac{\sqrt{2}\cos\alpha - \sqrt{2}\cos\alpha - \sqrt{2}\sin\alpha}{\cos\alpha + \sqrt{3}\sin\alpha - \sqrt{3}\sin\alpha} = \frac{-\sqrt{2}\sin\alpha}{\cos\alpha} = -\sqrt{2}tg\alpha;$$

4) $$\frac{\cos\alpha - 2\cos\left(\frac{\pi}{3} + \alpha\right)}{2\sin\left(\alpha - \frac{\pi}{6}\right) - \sqrt{3}\sin\alpha} = \frac{\cos\alpha - 2\left(\frac{1}{2}\cos\alpha - \frac{\sqrt{3}}{2}\sin\alpha\right)}{2\left(\frac{\sqrt{3}}{2}\sin\alpha + \frac{1}{2}\cos\alpha\right) - \sqrt{3}\sin\alpha} =$$

$$= \frac{\cos\alpha - \cos\alpha + \sqrt{3}\sin\alpha}{\sqrt{3}\sin\alpha - \cos\alpha - \sqrt{3}\sin\alpha} = \frac{\sqrt{3}\sin\alpha}{-\cos\alpha} = -\sqrt{3}tg\alpha.$$

305.

1) $\cos 6x \cdot \cos 5x + \sin 6x \cdot \sin 5x = -1;$
$\cos(6x - 5x) = -1$. Тогда $\cos x = -1;$
$x = \pi + 2\pi n,\ n \in \wedge;$
2) $\sin 3x \cdot \cos 5x - \sin 5x \cdot \cos 3x = -1;$
$\sin(3x - 5x) = -1;\ -\sin 2x = -1;$
$\sin 2x = 1$. Значит, $2x = \frac{\pi}{2} + 2\pi n;$

$x = \frac{\pi}{4} + 2\pi n, n \in \wedge;$

3) $\sqrt{2}\cos\left(\frac{\pi}{4} + x\right) - \cos x = 1;$

$\sqrt{2}\left(\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x\right) - \cos x = 1;$

$\cos x - \sin x - \cos x = 1;$

$\sin x = -1$. Поэтому $x = -\frac{\pi}{2} + 2\pi n,\ n \in \wedge;$

4) $\sqrt{2}\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)+\sin\frac{x}{2}=1$; $\sqrt{2}\left(\frac{\sqrt{2}}{2}\cos\frac{x}{2}-\frac{\sqrt{2}}{2}\sin\frac{x}{2}\right)+\sin\frac{x}{2}=1$;

$\cos\frac{x}{2}-\sin\frac{x}{2}+\sin\frac{x}{2}=1$;

$\cos\frac{x}{2}=1$.

Значит, $\frac{x}{2}=2\pi n$ и $x = 4\pi n$, $n \in \wedge$.

306.

1) $\frac{\mathrm{tg}29° + \mathrm{tg}31°}{1-\mathrm{tg}29°\cdot\mathrm{tg}31°} = \mathrm{tg}(29°+31°) = \mathrm{tg}60° = \sqrt{3}$;

2) $\frac{\mathrm{tg}\frac{7\pi}{16}-\mathrm{tg}\frac{3\pi}{16}}{1+\mathrm{tg}\frac{7\pi}{16}\cdot\mathrm{tg}\frac{3\pi}{16}} = \mathrm{tg}\left(\frac{7\pi}{16}-\frac{3\pi}{16}\right) = \mathrm{tg}\frac{\pi}{4} = 1$.

307.

1) $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\sin\alpha\cos\beta-\cos\alpha\sin\beta} = \frac{\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}+\frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}-\frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}} = \frac{\mathrm{tg}\alpha+\mathrm{tg}\beta}{\mathrm{tg}\alpha-\mathrm{tg}\beta}$;

2) $\frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)} = \frac{\cos\alpha\cos\beta+\sin\alpha\sin\beta}{\mathrm{ctg}\alpha\cdot\mathrm{ctg}\beta-1} = \frac{\frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta}+\frac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}}{\frac{\cos\alpha\cos\beta}{\sin\alpha\sin\beta}-\frac{\sin\alpha\sin\beta}{\sin\alpha\sin\beta}} = \frac{\mathrm{ctg}\alpha\cdot\mathrm{ctg}\beta+1}{\mathrm{ctg}\alpha\cdot\mathrm{ctg}\beta-1}$.

308.

1) $2\sin15°\cos15° = \sin2\cdot15° = \sin30° = \frac{1}{2}$;

2) $\cos^2 15° - \sin^2 15° = \cos2\cdot15° - \cos30° = \frac{\sqrt{3}}{2}$;

3) $(\cos75° - \sin75°)^2 = \cos^2 75° - 2\sin75°\cos75° + \sin^2 75° =$
$= 1 - \sin150° = 1 - \sin30° = 1 - \frac{1}{2} = \frac{1}{2}$;

4) $(\cos15° + \sin15°)^2 = \cos^2 15° + 2\sin15°\cos15° + \sin^2 15° =$
$= 1 + \sin30° = 1 + \frac{1}{2} = \frac{3}{2}$.

309.

1) $2\sin\frac{\pi}{8}\cos\frac{\pi}{8}=\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}$; 2) $\cos^2\frac{\pi}{8}-\sin^2\frac{\pi}{8}=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$;

3) $\sin\frac{\pi}{8}\cos\frac{\pi}{8}+\frac{1}{4}=\frac{1}{2}\sin\frac{\pi}{4}+\frac{1}{4}=\frac{\sqrt{2}}{4}+\frac{1}{4}=\frac{\sqrt{2}+1}{4}$;

4) $\frac{\sqrt{2}}{2}-\left(\cos\frac{\pi}{8}+\sin\frac{\pi}{8}\right)^2=\frac{\sqrt{2}}{2}-\left(1+2\sin\frac{\pi}{8}\cos\frac{\pi}{8}\right)=$

$=\frac{\sqrt{2}}{2}-\left(1+\sin\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}-\left(1+\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{2}}{2}-1-\frac{\sqrt{2}}{2}=-1$.

310.

1) Т.к. $\frac{\pi}{2}<\alpha<\pi$, то $\cos\alpha<0$, тогда

$\cos\alpha=-\sqrt{1-\sin^2\alpha}=-\sqrt{1-\frac{9}{25}}=-\frac{4}{5}$;

$\sin2\alpha = 2\sin\alpha\cos\alpha = 2\cdot\frac{3}{5}\left(-\frac{4}{5}\right)=-\frac{24}{25}$.

2) Т.к. $\pi<\alpha<\frac{3\pi}{2}$, то

$\sin\alpha<0$, тогда $\sin\alpha=-\sqrt{1-\frac{16}{25}}=-\frac{3}{5}$;

$\sin2\alpha = 2\sin\alpha\cos\alpha = 2\cdot\left(-\frac{3}{5}\right)\cdot\left(-\frac{4}{5}\right)=-\frac{24}{25}$.

311.

1) $\sin^2\alpha = 1-\cos^2\alpha$;

$\sin^2\alpha = 1-\frac{16}{25}=\frac{9}{25}$.

Т.к. $\cos2\alpha=\cos^2\alpha-\sin^2\alpha$, то

$\cos2\alpha=\frac{16}{25}-\frac{9}{25}=\frac{7}{25}$;

2) $\cos^2\alpha = 1-\sin^2\alpha$;

$\cos^2\alpha = 1-\frac{9}{25}=\frac{16}{25}$.

Т.к. $\cos2\alpha=\cos^2\alpha-\sin^2\alpha$, то

$\cos2\alpha=\frac{16}{25}-\frac{9}{25}=\frac{7}{25}$.

312.

1) $\sin\alpha\cos\alpha=\frac{2\sin\alpha\cos\alpha}{2}=\frac{\sin2\alpha}{2}$;

2) $\cos\alpha\cos\left(\frac{\pi}{2}-\alpha\right)=\sin\alpha\cos\alpha=\frac{\sin2\alpha}{2}$;

3) $\cos 4\alpha + \sin^2 2\alpha = \cos^2 2\alpha - \sin^2 2\alpha + \sin^2 2\alpha = \cos^2 2\alpha$;

4) $\sin 2\alpha + (\sin\alpha - \cos\alpha)^2 = 2\sin\alpha\cos\alpha + \sin^2\alpha - 2\sin\alpha\cos\alpha + \cos^2\alpha = 1$.

313.

1) $\dfrac{\cos 2\alpha + 1}{2\cos\alpha} = \dfrac{\cos^2\alpha - \sin^2\alpha + \cos^2\alpha + \sin^2\alpha}{2\cos\alpha} = \dfrac{2\cos^2\alpha}{2\cos\alpha} = \cos\alpha$;

2) $\dfrac{\sin 2\alpha}{1 - \cos^2\alpha} = \dfrac{2\sin\alpha\cos\alpha}{\sin^2\alpha} = \dfrac{2\cos\alpha}{\sin\alpha} = 2\mathrm{ctg}\alpha$;

3) $\dfrac{\sin^2\alpha}{(\sin\alpha + \cos\alpha)^2 - 1} = \dfrac{\sin^2\alpha}{\sin^2\alpha + 2\sin\alpha\cos\alpha + \cos^2\alpha - 1} =$

$= \dfrac{\sin^2\alpha}{2\sin\alpha\cos\alpha} = \dfrac{1}{2} tg\alpha$;

4) $\dfrac{1 + \cos 2\alpha}{1 - \cos 2\alpha} = \dfrac{\cos^2\alpha + \sin^2\alpha + \cos^2\alpha - \sin^2\alpha}{\cos^2\alpha + \sin^2\alpha - \cos^2\alpha + \sin^2\alpha} = \dfrac{2\cos^2\alpha}{2\sin^2\alpha} = \mathrm{ctg}^2\alpha$.

314.

1) $(\sin\alpha + \cos\alpha)^2 - 1 = 1 + 2\sin\alpha\cos\alpha - 1 = 2\sin\alpha\cos\alpha = \sin 2\alpha$;

2) $(\sin\alpha - \cos\alpha)^2 = \sin^2\alpha - 2\sin\alpha\cos\alpha + \cos^2\alpha = 1 - \sin 2\alpha$;

3) $\cos^4\alpha - \sin^4\alpha = (\cos^2\alpha - \sin^2\alpha)(\cos^2\alpha + \sin^2\alpha) = \cos 2\alpha$;

4) $2\cos^2\alpha - \cos 2\alpha = 2\cos^2\alpha - \cos^2\alpha + \sin^2\alpha = \cos^2\alpha + \sin^2\alpha = 1$.

315.

1) $\sin\alpha + \cos\alpha = \dfrac{1}{2}$.

Возведем в квадрат.

Получим: $(\sin\alpha + \cos\alpha)^2 = \dfrac{1}{4}$;

$1 + 2\sin\alpha\cos\alpha = \dfrac{1}{4}$; $\sin 2\alpha = \dfrac{1}{4} - 1 = -\dfrac{3}{4}$.

2) $\sin\alpha - \cos\alpha = -\dfrac{1}{3}$.

Возведем в квадрат

$(\sin\alpha - \cos\alpha)^2 = \dfrac{1}{9}$; $1 - 2\sin\alpha\cos\alpha = \dfrac{1}{9}$; $\sin 2\alpha = 1 - \dfrac{1}{9} = \dfrac{8}{9}$.

316.

1) $1 + \cos 2\alpha = \sin^2\alpha + \cos^2\alpha + \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha$;

2) $2\sin^2\alpha = \sin^2\alpha + \cos^2\alpha - \cos^2\alpha + \sin^2\alpha = 1 - \cos 2\alpha$.

317.

1) $2\cos^2 15° - 1 = 2\cos^2 15° - (\sin^2 15° + \cos^2 15°) =$

$= \cos^2 15° - \sin^2 15° = \cos 30° = \dfrac{\sqrt{3}}{2}$;

2) $1 - \sin^2 22{,}5° = \sin^2 22{,}5° + \cos^2 22{,}5° - 2\sin^2 22{,}5° =$

$= \cos^2 22{,}5° - \sin^2 22{,}5° = \cos 45° = \dfrac{\sqrt{2}}{2}$;

3) $2\cos^2\dfrac{\pi}{8} - 1 = 2\cos^2\dfrac{\pi}{8} - \left(\cos^2\dfrac{\pi}{8} + \sin^2\dfrac{\pi}{8}\right) = \cos^2\dfrac{\pi}{8} - \sin^2\dfrac{\pi}{8} =$

$= \cos\dfrac{\pi}{4} = \dfrac{\sqrt{2}}{2}$;

4) $1 - 2\sin^2\dfrac{\pi}{12} = \cos^2\dfrac{\pi}{12} + \sin^2\dfrac{\pi}{12} - 2\sin^2\dfrac{\pi}{12} = \cos^2\dfrac{\pi}{12} - \sin^2\dfrac{\pi}{12} =$

$= \cos\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2}$.

318.

1) $1 - 2\sin^2 5\alpha = \sin^2 5\alpha + \cos^2 5\alpha - 2\sin^2 5\alpha = \cos^2 5\alpha - \sin^2 5\alpha = \cos 10\alpha$;

2) $2\cos^2 3\alpha - 1 = 2\cos^2 3\alpha - (\sin^2 3\alpha + \cos^2 3\alpha) =$
$= \cos^2 3\alpha - \sin^2 3\alpha = \cos 6\alpha$;

3) $\dfrac{1-\cos 2\alpha}{\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}} = \dfrac{\sin^2\alpha + \cos^2\alpha - \cos^2\alpha + \sin^2\alpha}{\frac{1}{2}\sin\alpha} = \dfrac{4\sin^2\alpha}{\sin\alpha} = 4\sin\alpha$;

4) $\dfrac{2\cos^2\frac{\alpha}{2} - 1}{\sin 2\alpha} = \dfrac{2\cos^2\frac{\alpha}{2} - \cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}}{2\sin\alpha\cdot\cos\alpha} = \dfrac{\cos\alpha}{2\sin\alpha\cos\alpha} = \dfrac{1}{2\sin\alpha}$.

319.

1) $\dfrac{\cos 2\alpha}{\sin\alpha\cos\alpha + \sin^2\alpha} = \dfrac{(\cos\alpha - \sin\alpha)(\cos\alpha + \sin\alpha)}{\sin\alpha(\cos\alpha + \sin\alpha)} = \dfrac{\cos\alpha - \sin\alpha}{\sin\alpha} =$

$= \dfrac{\cos\alpha}{\sin\alpha} - \dfrac{\sin\alpha}{\sin\alpha} = ctg - 1$;

2) $\dfrac{\sin 2\alpha - 2\cos\alpha}{\sin\alpha - \sin^2\alpha} = \dfrac{2\cos\alpha(\sin\alpha - 1)}{\sin\alpha(1 - \sin\alpha)} = -\dfrac{2\cos\alpha}{\sin\alpha} = -2\mathrm{ctg}\alpha$;

3) $tg\alpha\cdot(1 + \cos 2\alpha) = tg\alpha\cdot(\cos^2\alpha + \sin^2\alpha + \cos^2\alpha - \sin^2\alpha) =$

$= \dfrac{\sin\alpha}{\cos\alpha}\cdot 2\cos^2\alpha = 2\sin\alpha\cos\alpha = \sin 2\alpha$;

4) $\dfrac{1-\cos 2\alpha+\sin 2\alpha}{1+\cos 2\alpha+\sin 2\alpha}=$

$$=\frac{\cos^2\alpha+\sin^2\alpha-\cos^2\alpha+\sin^2\alpha+2\sin\alpha\cos\alpha}{\cos^2\alpha+\sin^2\alpha+\cos^2\alpha-\sin^2\alpha+2\sin\alpha\cos\alpha}\cdot\frac{\cos\alpha}{\sin\alpha}=$$

$$=\frac{2\sin\alpha(\sin\alpha+\cos\alpha)\cdot\cos\alpha}{2\cos\alpha(\sin\alpha+\cos\alpha)\cdot\sin\alpha}=1.$$

320.

1) $\sin 2x-2\cos x=0$;

$2\cos x\cdot\sin x-2\cos x=0$;

$2\cos x\,(\sin x-1)=0$.

Тогда $\left[\begin{array}{l}\cos x=0\\ \sin x-1=0\end{array}\right.$; $\left[\begin{array}{l}x=\dfrac{\pi}{2}+\pi n\\ \sin x=1\end{array}\right.$; $\left[\begin{array}{ll}x=\dfrac{\pi}{2}+\pi n, & n\in Z\\ x=\dfrac{\pi}{2}+2\pi n, & n\in Z\end{array}\right.$.

Ответ: $\dfrac{\pi}{2}+\pi n$.

2) $\cos 2x+3\sin x=1$;

$\cos^2x-\sin^2x+3\sin x-\sin^2x-\cos^2x=0$;

$3\sin x-2\sin^2x=0$;

$\sin x\,(-2\sin x+3)=0$;

$\left[\begin{array}{l}\sin x=0\\ -2\sin x+3=0\end{array}\right.$; $\left[\begin{array}{ll}x=\pi n, & n\in Z\\ \sin x=1{,}5 & -\text{нет решения}\end{array}\right.$.

Ответ: πn; $n\in\wedge$.

3) $2\sin x=\sin 2x$;

$2\sin x-2\sin x\cdot\cos x=0$;

$2\sin x\,(1-\cos x)=0$;

$\left[\begin{array}{l}\sin x=0\\ 1-\cos x=0\end{array}\right.$; $\left[\begin{array}{ll}x=\pi n, & n\in Z\\ \cos x=1\end{array}\right.$; $\left[\begin{array}{ll}x=\pi n, & n\in Z\\ x=2\pi n\end{array}\right.$.

Ответ: πn.

4) $\sin^2x=-\cos 2x$;

$\sin^2x+\cos^2x-\sin^2x=0$;

$\cos^2x=0$;

$\cos x=0$.

Ответ: $\dfrac{\pi}{2}+\pi n$; $n\in\wedge$.

321.

Т.к. $\operatorname{tg}2\alpha = \dfrac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha}$, то $\operatorname{tg}2\alpha = \dfrac{2\cdot 0{,}6}{1-0{,}36} = \dfrac{1{,}2}{0{,}64} = \dfrac{120}{64} = 1\dfrac{7}{8}$.

322.

1) $\dfrac{2\operatorname{tg}\dfrac{\pi}{8}}{1-\operatorname{tg}^2\dfrac{\pi}{8}} = \operatorname{tg}\dfrac{\pi}{4} = 1$; 2) $\dfrac{6\operatorname{tg}15^\circ}{1-\operatorname{tg}^2 15^\circ} = 3\cdot\operatorname{tg}30^\circ = 3\cdot\dfrac{1}{\sqrt{3}} = \sqrt{3}$.

323.

1) $\sin\dfrac{13}{2}\pi = \sin\left(6\pi + \dfrac{\pi}{2}\right) = \sin\dfrac{\pi}{2} = 1$;

2) $\sin 17\pi = \sin(18\pi - \pi) = -\sin\pi = 0$;

3) $\cos 7\pi = \cos(8\pi - \pi) = \cos\pi = -1$;

4) $\cos\dfrac{11}{2}\pi = \cos\left(6\pi - \dfrac{\pi}{2}\right) = \cos\dfrac{\pi}{2} = 0$;

5) $\sin 720^\circ = \sin(2\cdot 360^\circ) = 0$;

6) $\cos 540^\circ = \cos(360^\circ + 180^\circ) = \cos 180^\circ = -1$.

324.

1) $\cos 420^\circ = \cos(360^\circ + 60^\circ) = \cos 60^\circ = \dfrac{1}{2}$;

2) $\operatorname{tg}570^\circ = \operatorname{tg}(3\cdot 180^\circ + 30^\circ) = \operatorname{tg}30^\circ = \dfrac{1}{\sqrt{3}}$;

3) $\sin 3630^\circ = \sin(10\cdot 360^\circ + 30^\circ) = \sin 30^\circ = \dfrac{1}{2}$;

4) $\operatorname{ctg}960^\circ = \operatorname{ctg}(5\cdot 180^\circ + 60^\circ) = \operatorname{ctg}60^\circ = \dfrac{1}{\sqrt{3}}$;

5) $\sin\dfrac{13\pi}{6} = \sin\left(2\pi + \dfrac{\pi}{6}\right) = \sin\dfrac{\pi}{6} = \dfrac{1}{2}$;

6) $\operatorname{tg}\dfrac{11\pi}{6} = \operatorname{tg}\left(2\pi - \dfrac{\pi}{6}\right) = -\operatorname{tg}\dfrac{\pi}{6} = -\dfrac{1}{\sqrt{3}}$.

325.

1) $\cos 150^\circ = \cos(90^\circ + 60^\circ) = -\sin 60^\circ = -\dfrac{\sqrt{3}}{2}$;

2) $\sin 135^\circ = \sin(90^\circ + 45^\circ) = \cos 45^\circ = \dfrac{\sqrt{2}}{2}$;

3) cos120° = – cos60° = – $\frac{1}{2}$;

4) sin315° = sin (360° – 45°) = – 45° = – $\frac{\sqrt{2}}{2}$.

326.

1) $\mathrm{tg}\frac{5\pi}{4}=\mathrm{tg}\left(\pi+\frac{\pi}{4}\right)=\mathrm{tg}\frac{\pi}{4}=1$;

2) $\sin\frac{7\pi}{6}=\sin\left(\pi+\frac{\pi}{6}\right)=-\sin\frac{\pi}{6}=-\frac{1}{2}$;

3) $\cos\frac{5\pi}{3}=\cos\left(2\pi-\frac{\pi}{3}\right)=\cos\frac{\pi}{3}=\frac{1}{2}$;

4) $\sin\left(-\frac{11\pi}{6}\right)=-\sin\left(2\pi-\frac{\pi}{6}\right)=\sin\frac{\pi}{6}=\frac{1}{2}$;

5) $\cos\left(-\frac{7\pi}{3}\right)=\cos\left(2\pi+\frac{\pi}{3}\right)=\cos\frac{\pi}{3}=\frac{1}{2}$;

6) $\mathrm{tg}\left(-\frac{2\pi}{3}\right)=-\mathrm{tg}\left(\pi-\frac{\pi}{3}\right)=\mathrm{tg}\frac{\pi}{3}=\sqrt{3}$.

327.

1) cos630° – sin1470° – ctg1125° = cos(720° – 90°) – – sin(1440° + 30°) – ctg(1080° + 45°) = cos90° – sin30° – ctg45° = = 0 – $\frac{1}{2}$ – 1 = – $\frac{3}{2}$;

2) tg1800° – sin495° + cos945° = 0 – sin135° + cos225° = =–sin(90° + 45°)+cos(180° + 45°)= –cos45° –cos45° = – 2· $\frac{\sqrt{2}}{2}$ = – $\sqrt{2}$;

3) $\sin(-7\pi)-2\cos\frac{31\pi}{3}-tg\,\frac{7\pi}{4}=-\sin(6\pi+\pi)-$

$-2\cos\left(10\pi+\frac{\pi}{3}\right)-tg\left(2\pi-\frac{\pi}{4}\right)=-\sin\pi-2\cos\frac{\pi}{3}+tg\,\frac{\pi}{4}=$

$=0-2\cdot\frac{1}{2}+1=-1+1=0$;

4) $\cos(-9\pi)+2\sin\left(-\frac{49\pi}{6}\right)-ctg\left(-\frac{21\pi}{4}\right)=\cos\pi-2\sin\left(8\pi+\frac{\pi}{6}\right)+$

$+ctg\left(5\pi+\frac{\pi}{4}\right)=-1-\sin\frac{\pi}{6}+ctg\,\frac{\pi}{4}=-1-2\cdot\frac{1}{2}+1=-1-1+1=-1$.

328.

1) $\cos^2(\pi - \alpha) + \sin^2(\alpha - \pi) = \cos^2\alpha + \sin^2\alpha = 1$;

2) $\cos(\pi - \alpha)\cos(3\pi - \alpha) - \sin(\alpha - \pi)\sin(\alpha - 3\pi) =$
$= \cos(\pi - \alpha)\cos(3\pi - \alpha) - \sin(\pi - \alpha)\sin(3\pi - \alpha) = \cos(\pi - \alpha + 3\pi - \alpha) =$
$= \cos(4\pi - 2\alpha) = \cos 2\alpha$.

329.

1) $\cos 723° + \sin 900° = \cos(360°\cdot 20 + 30°) + \sin(360°\cdot 2 + 180°) =$

$$= \cos 30° + \sin 180° = \frac{\sqrt{3}}{2} + 0 = \frac{\sqrt{3}}{2};$$

2) $\sin 300° + tg 150° = \sin(360° - 60°) + tg(180° - 30°) =$

$$= -\sin 60° - tg 30° = -\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{3} = \frac{-5\sqrt{3}}{6};$$

3) $$2\sin 6{,}5\pi - \sqrt{3}\sin\frac{19\pi}{3} = 2\sin\left(6\pi + \frac{\pi}{2}\right) - \sqrt{3}\sin\left(6\pi + \frac{\pi}{3}\right) =$$

$$= 2\sin\frac{\pi}{2} - \sqrt{3}\sin\frac{\pi}{3} = 2 - \sqrt{3}\cdot\frac{\sqrt{3}}{2} = 2 - \frac{3}{2} = \frac{1}{2};$$

4) $$\sqrt{2}\cos 4{,}25\pi - \frac{1}{\sqrt{3}}\cos\frac{61\pi}{6} = \sqrt{2}\cos\left(4\pi + \frac{\pi}{4}\right) - \frac{1}{\sqrt{3}}\cos\left(10\pi + \frac{\pi}{6}\right) =$$

$$= \sqrt{2}\cos\frac{\pi}{4} - \frac{1}{\sqrt{3}}\cos\frac{\pi}{6} = \sqrt{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{2} = 1 - \frac{1}{2} = \frac{1}{2};$$

5) $$\frac{\sin(-6{,}5\pi) + tg(-7\pi)}{\cos(-7\pi) + ctg(-16{,}25\pi)} = \frac{-\sin\left(6\pi + \frac{\pi}{2}\right) - tg(6\pi + \pi)}{\cos(6\pi + \pi) - ctg\left(16\pi + \frac{\pi}{4}\right)} =$$

$$= \frac{-\sin\frac{\pi}{2} - tg\pi}{\cos\pi - ctg\frac{\pi}{4}} = \frac{-1 - 0}{-1 - 1} = \frac{1}{2};$$

6) $$\frac{\cos(-540°) + \sin 480°}{tg 405° - ctg 330°} = \frac{\cos(720° - 180°) + \sin(360° + 120°)}{tg(360° + 45°) - ctg(360° - 30°)} =$$

$$= \frac{\cos 180° + \sin 120°}{tg 45° + ctg 30°} = \frac{-1 + \frac{\sqrt{3}}{2}}{1 + \sqrt{3}} = \frac{\sqrt{3} - 2}{2(1 + \sqrt{3})} = \frac{(\sqrt{3} - 2)(1 - \sqrt{3})}{2(1 + \sqrt{3})(1 - \sqrt{3})} = \frac{5 - 3\sqrt{3}}{4}.$$

330.

1) $\dfrac{\sin\left(\dfrac{\pi}{2}-\alpha\right)+\sin(\pi-\alpha)}{\cos(\pi-\alpha)+\sin(2\pi-\alpha)}=\dfrac{\cos\alpha+\sin\alpha}{-\cos\alpha-\sin\alpha}=-1$;

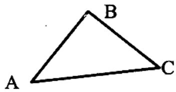
2) $\dfrac{\cos(\pi-\alpha)+\cos\left(\dfrac{\pi}{2}-\alpha\right)}{\sin(\pi-\alpha)-\sin\left(\dfrac{\pi}{2}-\alpha\right)}=\dfrac{-\cos\alpha+\sin\alpha}{\sin\alpha-\cos\alpha}=1$;

3) $\dfrac{\sin(\alpha-\pi)}{\text{tg}(\alpha+\pi)}\cdot\dfrac{\text{tg}(\pi-\alpha)}{\cos\left(\dfrac{\pi}{2}-\alpha\right)}=\dfrac{-\sin\alpha\cdot(-\text{tg}\alpha)}{\text{tg}\alpha\cdot\sin\alpha}=1$;

4) $\dfrac{\sin^2(\pi-\alpha)+\sin\left(\dfrac{\pi}{2}-\alpha\right)}{\sin(\pi-\alpha)}\cdot tg(\pi-\alpha)=\dfrac{\sin^2\alpha+\cos^2\alpha}{\sin\alpha}\cdot(-tg\alpha)=$

$=\dfrac{1}{\sin\alpha}\cdot\left(-\dfrac{\sin\alpha}{\cos\alpha}\right)=-\dfrac{1}{\cos\alpha}$.

331.



Пусть α, β, γ – углы треугольника,
$\sin\gamma = \sin(180° - (\alpha + \beta)) = \sin180°\cdot\cos(\alpha + \beta) - \cos180° \cdot \sin(\alpha + \beta) = 0\cdot\cos(\alpha + \beta) - (-1)\cdot\sin(\alpha + \beta) = \sin(\alpha + \beta)$.

332.

1) $\sin\left(\dfrac{\pi}{2}+\alpha\right)=\sin\dfrac{\pi}{2}\cdot\cos\alpha+\cos\dfrac{\pi}{2}\cdot\sin\alpha=$
$=1\cdot\cos\alpha+0\cdot\sin\alpha=\cos\alpha$;

2) $\cos\left(\dfrac{\pi}{2}+\alpha\right)=\cos\dfrac{\pi}{2}\cdot\cos\alpha-\sin\dfrac{\pi}{2}\cdot\sin\alpha=$
$=0\cdot\cos\alpha-1\cdot\sin\alpha=-\sin\alpha$;

3) $\cos\left(\dfrac{3\pi}{2}-\alpha\right)=\cos\dfrac{3\pi}{2}\cdot\cos\alpha+\sin\dfrac{3\pi}{2}\cdot\sin\alpha=$
$=0\cdot\cos\alpha+(-1)\cdot\sin\alpha=-\sin\alpha$;

4) $\sin\left(\dfrac{3\pi}{2}-\alpha\right)=\sin\dfrac{3\pi}{2}\cdot\cos\alpha-\cos\dfrac{3\pi}{2}\cdot\sin\alpha=$
$=-1\cdot\cos\alpha-0\cdot\sin\alpha=-\cos\alpha$.

333.

1) $\cos\left(\frac{\pi}{2}-x\right)=1$;

$\sin x = 1$.

Тогда $x=\frac{\pi}{4}+2\pi n, \quad n\in Z$.

2) $\sin(\pi - x) = 1$;

$\sin x = 1$.

Значит $x=\frac{\pi}{4}+2\pi n, \qquad n\in Z$.

3) $\cos(x-\pi) = 0$;

$\cos x = 0$.

Поэтому $x=\frac{\pi}{4}+2\pi n, \quad n\in Z$.

4) $\sin\left(x-\frac{\pi}{2}\right)=1$;

$-\cos x = 1$;

$\cos x = -1$.

Тогда $x = \pi + 2\pi n$, $n \in Z$.

334.

1) $\sin\left(\frac{\pi}{4}+\alpha\right)-\cos\left(\frac{\pi}{4}-\alpha\right)=$

$$=\frac{\sqrt{2}}{2}\cos\alpha+\frac{\sqrt{2}}{2}\sin\alpha-\frac{\sqrt{2}}{2}\cos\alpha-\frac{\sqrt{2}}{2}\sin\alpha=0;$$

2) $\cos\left(\frac{\pi}{6}-\alpha\right)-\sin\left(\frac{\pi}{3}+\alpha\right)=$

$$=\frac{\sqrt{3}}{2}\cos\alpha+\frac{1}{2}\sin\alpha-\frac{\sqrt{3}}{2}\cos\alpha-\frac{1}{2}\sin\alpha=0.$$

336.

1) I четв.;

2) III четв.;

3) III четв.;

4) IV четв.;

5) II четв.

337.

1) $\sin 3\pi = 0$; $\cos 3\pi = -1$;

2) $\sin 4\pi = 0$; $\cos 4\pi = 1$;

3) $\sin 3{,}5\pi = -1$;

$\cos 3{,}5\pi = 0$;

4) $\sin\left(\frac{5\pi}{2}\right)=\sin\frac{\pi}{2}=1$;

$\cos\frac{5\pi}{2}=0$;

5) $\sin \pi n = 0$;

$\cos\pi n = \begin{cases} 1, & n - \text{четное} \\ -1, & n - \text{нечетное} \end{cases}$;

6) $\sin((2n+1)\pi) = 0$;

$\cos((2n+1)\pi) = -1$, $n \in Z$.

338.

1) $\sin 3\pi - \cos\frac{3\pi}{2} = 0 - 0 = 0;$

2) $\cos 0 - \cos 3\pi + \cos 3{,}5\pi = 1 - (-1) + 0 = 2;$

3) $\sin\pi k + \cos 2\pi k = 0 + 1 = 1;$

4) $\cos\frac{(2k+1)\pi}{2} - \sin\frac{(4k+1)\pi}{2} = 0 - 1 = -1.$

339.

1) Т.к. $\frac{\pi}{2} < \alpha < \pi$, то $\cos\alpha < 0$, тогда

$\cos\alpha = -\sqrt{1-\sin^2\alpha} = -\sqrt{1-\frac{1}{3}} = -\frac{\sqrt{6}}{3}.$

2) Т.к. $\pi < \alpha < \frac{3\pi}{2}$, то $\mathrm{tg}\alpha < 0$,

т.к. $1 + \mathrm{tg}^2\alpha = \frac{1}{\cos^2\alpha}$, то $\mathrm{tg}\alpha = \sqrt{\frac{1}{\cos^2\alpha} - 1} = \sqrt{\frac{9}{5} - 1} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}.$

3) Т.к. $0 < \alpha < \frac{\pi}{2}$, то $\sin\alpha > 0$, $\mathrm{ctg}\alpha = \frac{1}{\mathrm{tg}\alpha} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4};$

$1 + \mathrm{ctg}^2\alpha = \frac{1}{\sin^2\alpha}$ $\sin\alpha = \sqrt{\frac{1}{1+\mathrm{ctg}^2\alpha}} = \sqrt{\frac{1}{1+\frac{1}{8}}} = \sqrt{\frac{8}{9}} = \sqrt{\frac{2\sqrt{2}}{3}}.$

4) Т.к. $\pi < \alpha < \frac{3\pi}{2}$, то $\cos\alpha < 0$,

$1 + \mathrm{tg}^2\alpha = \frac{1}{\cos^2\alpha}$, $\quad \mathrm{tg}\alpha = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2};$

$\cos\alpha = -\sqrt{\frac{1}{1+\mathrm{ctg}^2\alpha}} = \sqrt{\frac{1}{1+\frac{1}{2}}} = -\sqrt{\frac{2}{3}} = \frac{-\sqrt{6}}{3}.$

340.

1) $5\sin^2\alpha + \mathrm{tg}\alpha \cdot \cos\alpha + 5\cos^2\alpha =$

$= 5(\sin^2\alpha + \cos^2\alpha) + \frac{\sin\alpha}{\cos\alpha}\cdot\cos\alpha = 5 + \sin\alpha;$

2) $\mathrm{ctg}\alpha \cdot \sin\alpha - 2\cos^2\alpha - 2\sin^2\alpha =$

$= \frac{\sin\alpha}{\cos\alpha}\cdot\sin\alpha - 2(\sin^2\alpha + \cos^2\alpha) = \cos\alpha - 2;$

3) $\frac{3}{1+\operatorname{tg}^2\alpha} = 3\cos^2\alpha$. Т.к. $\cos^2\alpha = \frac{1}{1+\operatorname{tg}^2\alpha}$;

4) $\frac{5}{1+\operatorname{ctg}^2\alpha} = 5\sin^2\alpha$. Т.к. $\sin^2\alpha = \frac{1}{1+\operatorname{ctg}^2\alpha}$.

341.

1) $2\sin(-\alpha)\cdot\cos\left(\frac{\pi}{2}-\alpha\right) - 2\cos(-\alpha)\cdot\sin\left(\frac{\pi}{2}-\alpha\right) =$

$= -2\sin\alpha\cdot\sin\alpha - 2\cos\alpha\cdot\cos\alpha = -2\sin^2\alpha - 2\cos^2\alpha =$

$= -2(\sin^2\alpha + \cos^2\alpha) = -2$;

2) $3\sin(\pi-\alpha)\cos\left(\frac{\pi}{2}-\alpha\right) + 3\sin^2\left(\frac{\pi}{2}-\alpha\right) =$

$= 3\sin\alpha\cdot\sin\alpha + 3\cos^2\alpha = 3(\sin^2\alpha + \cos^2\alpha) = 3$;

3) $(1-\operatorname{tg}(-\alpha))\cdot(1-\operatorname{tg}(\pi+\alpha))\cos^2\alpha = (1+\operatorname{tg}\alpha)(1-\operatorname{tg}\alpha)\cdot\cos^2\alpha =$

$= (1-\operatorname{tg}^2\alpha)\cdot\cos^2\alpha = \cos^2\alpha - \sin^2\alpha = \cos 2\alpha$;

4) $(1+\operatorname{tg}^2(-\alpha))\cdot\left(\frac{1}{1+\operatorname{ctg}^2(-\alpha)}\right) = (1+\operatorname{tg}^2\alpha)\cdot\frac{1}{1+\operatorname{ctg}^2\alpha} =$

$= \frac{(1+\operatorname{tg}^2\alpha)\cdot\operatorname{tg}^2\alpha}{1+\operatorname{tg}^2\alpha} = \operatorname{tg}^2\alpha$.

342.

1) $\sin\left(\frac{3\pi}{2}-\alpha\right)+\sin\left(\frac{3\pi}{2}+\alpha\right)= \quad \cos\alpha - \cos\alpha = -2\cos\alpha$.

Т.к. $\cos\alpha = \frac{1}{4}$, то значение выражения равно $-\frac{1}{2}$.

2) $\cos\left(\frac{\pi}{2}+\alpha\right)+\cos\left(\frac{3\pi}{2}-\alpha\right) = -\sin\alpha - \sin\alpha = -2\sin\alpha$.

Т.к. $\sin\alpha = \frac{1}{6}$, то значение выражения равно $-\frac{1}{3}$.

343.

1) $2\sin 75^\circ\cdot\cos 75^\circ = \sin 150^\circ = \sin(180^\circ - 150^\circ) = \sin 30^\circ = \frac{1}{2}$;

2) $\cos^2 75^\circ - \sin^2 75^\circ = \cos 150^\circ = -\cos(180^\circ - 150^\circ) =$

$= -\cos 30^\circ = -\frac{\sqrt{3}}{2}$;

3) $\sin 15° = \sin(45° - 30°) = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{2}(\sqrt{3}-1)}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$;

4) $\sin 75° = \sin(45° + 30°) = \frac{\sqrt{2}\cdot\sqrt{3}}{2\cdot 2} + \frac{\sqrt{2}}{2\cdot 2} = \frac{\sqrt{2}(\sqrt{3}+1)}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$.

344.

1) $\cos^2(\pi - \alpha) - \cos^2\left(\frac{\pi}{2} - \alpha\right) = \cos^2\alpha - \sin^2\alpha = \cos 2\alpha$;

2) $2\sin\left(\frac{\pi}{2} - \alpha\right)\cos\left(\frac{\pi}{2} - \alpha\right) = 2\cdot\cos\alpha\cdot\sin\alpha = \sin 2\alpha$;

3) $\frac{\cos^2(2\pi + \alpha) - \sin^2(2\pi + \alpha)}{2\cos(2\pi + \alpha)\cos\left(\frac{\pi}{2} - \alpha\right)} = \frac{\cos^2\alpha - \sin^2\alpha}{2\cos\alpha\sin\alpha} = \frac{\cos 2\alpha}{\sin 2\alpha} = \mathrm{ctg}2\alpha$;

4) $\frac{2\sin(\pi - \alpha)\sin\left(\frac{\pi}{2} - \alpha\right)}{\sin^2\left(\alpha - \frac{\pi}{2}\right) - \sin^2(\alpha - \pi)} = \frac{2\cos\alpha\sin\alpha}{\cos^2\alpha - \sin^2\alpha} = \frac{\sin 2\alpha}{\cos 2\alpha} = \mathrm{tg}2\alpha$.

345.

1) $\sin\frac{47\pi}{6} = \sin\left(8\pi - \frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$;

2) $\mathrm{tg}\frac{25\pi}{4} = \mathrm{tg}\left(6\pi + \frac{\pi}{4}\right) = \mathrm{tg}\frac{\pi}{4} = 1$;

3) $\mathrm{ctg}\frac{27\pi}{4} = \mathrm{ctg}\left(7\pi - \frac{\pi}{4}\right) = \mathrm{ctg}\left(-\frac{\pi}{4}\right) = \mathrm{ctg}\left(-\frac{\pi}{4}\right) = -1$;

4) $\cos\frac{21\pi}{4} = \cos\left(5\pi + \frac{\pi}{4}\right) = \cos\left(\pi + \frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$.

346.

1) $\cos\frac{23\pi}{4} - \sin\frac{15\pi}{4} = \cos\left(6\pi - \frac{\pi}{4}\right) - \sin\left(4\pi\ \frac{\pi}{4}\right) = \cos\left(-\frac{\pi}{4}\right) - \sin\left(-\frac{\pi}{4}\right) =$

$= \cos\frac{\pi}{4} + \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$;

2) $\sin\frac{25\pi}{3} - tg\frac{10\pi}{3} = \sin\left(8\pi + \frac{\pi}{3}\right) - tg\left(3\pi + \frac{\pi}{3}\right) = \sin\frac{\pi}{3} - tg\frac{\pi}{3} =$

$= \frac{\sqrt{3}}{2} - \sqrt{3} = -\frac{\sqrt{3}}{2}$;

3) $3\cos 3660° + \sin(-1560°) = 3\cos(10 \cdot 360° + 60°) +$
$+ \sin(-120° - 4 \cdot 360°) = 3 \cdot \cos 60° - \sin 120° = 3 \cdot \frac{1}{2} - \sin 60° =$
$= \frac{3}{2} - \frac{\sqrt{3}}{2} = \frac{3-\sqrt{3}}{2}$;

4) $\cos(-945°) + \mathrm{tg}1035° = \cos(-3 \cdot 360° + 135°) +$
$+ \mathrm{tg}(2{,}5 \cdot 360° + 135°) = \cos 135° + \mathrm{tg}135° = -\cos 45° - \mathrm{tg}45° =$
$= -\frac{\sqrt{2}}{2} - 1 = -\frac{2+\sqrt{2}}{2}$.

347.

1) $\sin 3 > \cos 4$,
т.к. $\sin 3 > 0$, $\cos 4 < 0$.

2) $\cos 0 > \sin 5$,
т.к. $\sin 5 < 0$, $\cos 0 = 1$.

348.

1) $\sin 3{,}5 \cdot \mathrm{tg}3{,}5 = \frac{\sin^2 3{,}5}{\cos 3{,}5} < 0$, т.к. $\sin^2 3{,}5>0$, $\cos 3{,}5<0$;

2) $\cos 5{,}01 \cdot \sin 0{,}73 > 0$, т.к. $\cos 5{,}01>0$, $\sin 0{,}73>0$;

3) $\frac{tg13}{\cos 15} < 0$, т.к. $\mathrm{tg}13>0$, $\cos 15<0$;

4) $\sin 1 \cdot \cos 2 \cdot \mathrm{tg}3 >0$, т.к. $\sin 1>0$, $\cos 2$ и $\mathrm{tg}3<0$.

349.

1) $\sin\frac{\pi}{8} \cdot \cos\frac{3\pi}{8} + \sin\frac{3\pi}{8} \cdot \cos\frac{\pi}{8} = \sin\left(\frac{\pi}{8} + \frac{3\pi}{8}\right) = \sin\frac{\pi}{2} = 1$;

2) $\sin 165° = \sin(120° + 45°) = \sin 120° \cdot \cos 45° + \cos 120° \cdot \sin 45° =$
$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}\left(\sqrt{3}-1\right)}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$;

3) $\sin 105° = \sin(60° + 45°) = \sin 60° \cdot \cos 45° + \cos 60° \cdot \sin 45° =$
$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}\left(\sqrt{3}+1\right)}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$;

4) $\sin\frac{\pi}{12} = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\frac{\pi}{3} \cdot \cos\frac{\pi}{4} - \cos\frac{\pi}{3} \cdot \sin\frac{\pi}{4} =$
$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}\left(\sqrt{3}-1\right)}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$;

5) $1 - \sin^2 195° = \cos^2 195° - \sin^2 195° = \cos 390° = \cos(360° + 30°) =$
$= \cos 30° = \frac{\sqrt{3}}{2}$;

б) $2\cos^2\frac{3\pi}{8}-1=2\cos^2\frac{3\pi}{8}-\cos^2\frac{3\pi}{8}-\sin^2\frac{3\pi}{8}=\cos^2\frac{3\pi}{8}-\sin^2\frac{3\pi}{8}=$

$=\cos\frac{3\pi}{4}=-\frac{\sqrt{2}}{2}$;

350.

1) $(1+\text{tg}(-\alpha))\cdot(1-\text{ctg}(-\alpha)-\frac{\sin(-\alpha)}{\cos(-\alpha)}=(1-\text{tg}\alpha)\cdot(1+\text{ctg}\alpha)+$

$+\text{tg}\alpha=1+\text{ctg}\alpha-\text{tg}\alpha-1+\text{tg}\alpha=\text{ctg}\alpha$;

2) $\frac{ctg\alpha+tg(-\alpha)}{\cos\alpha+\sin(-\alpha)}+\frac{tg(-\alpha)}{\sin\alpha}=\frac{ctg\alpha-tg\alpha}{\cos\alpha-\sin\alpha}-\frac{1}{\cos\alpha}=$

$=\frac{\cos^2\alpha-\sin^2\alpha}{\cos\alpha\cdot\sin\alpha(\cos\alpha-\sin\alpha)}-\frac{1}{\cos\alpha}=\frac{\cos\alpha+\sin\alpha}{\cos\alpha\cdot\sin\alpha}-\frac{1}{\cos\alpha}=$

$=\frac{\cos\alpha}{\cos\alpha\cdot\sin\alpha}=\frac{1}{\sin\alpha}$.

351.

Т.к. $\frac{\pi}{2}<\alpha<\pi$, то $\cos\alpha<0$, тогда $\cos\alpha=-\sqrt{1-\sin^2\alpha}=-\sqrt{1-\frac{5}{9}}=-\frac{2}{3}$;

$\text{tg}\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{\sqrt{5}/3}{-2/3}=-\frac{\sqrt{5}}{2}$; $\text{ctg}\,\alpha=\frac{1}{\text{tg}\alpha}=-\frac{2}{\sqrt{5}}$;

$\sin 2\alpha=2\sin\alpha\cos\alpha=2\cdot\frac{\sqrt{5}}{3}\cdot\left(-\frac{2}{3}\right)=-\frac{4\sqrt{5}}{9}$;

$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha=\frac{4}{9}-\frac{5}{9}=-\frac{1}{9}$;

352.

1) $\cos^3\alpha\cdot\sin\alpha-\sin^3\alpha\cdot\cos\alpha=\cos\alpha\cdot\sin\alpha(\cos^2\alpha-\sin^2\alpha)=$

$=\frac{1}{2}\sin2\alpha\cdot\cos2\alpha=\frac{1}{4}\sin4\alpha$;

2) $\frac{\sin\alpha+\sin2\alpha}{1+\cos\alpha+\cos2\alpha}=\frac{\sin\alpha(1+2\cos\alpha)}{2\cos^2\alpha+\cos\alpha}=\frac{\sin\alpha(1+2\cos\alpha)}{\cos\alpha(1+2\cos\alpha)}=tg\alpha$.

353.

1) $\frac{\sin2\alpha-\sin2\alpha\cdot\cos2\alpha}{4\cos\alpha}=\frac{\sin2\alpha(1-\cos2\alpha)}{4\cos\alpha}=\frac{2\sin\alpha\cos\alpha\cdot2\sin^2\alpha}{4\cos\alpha}=\sin^3\alpha$;

2) $\dfrac{2\cos^2 2\alpha}{\sin 4\alpha\cdot\cos 4\alpha+\sin 4\alpha}=\dfrac{2\cos^2 2\alpha}{\sin 4\alpha(\cos 4\alpha+1)}=$

$=\dfrac{2\cos^2 2\alpha}{2\sin 2\alpha\cos 2\alpha(2\cos^2 2\alpha)}=\dfrac{1}{2\sin 2\alpha\cos 2\alpha}=\dfrac{1}{\sin 4\alpha}$;

3) $\dfrac{\cos 2\alpha+\sin 2\alpha\cdot\cos 2\alpha}{2\sin^2\alpha-1}=\dfrac{\cos 2\alpha(1+\sin 2\alpha)}{\sin^2\alpha-\cos^2\alpha}=$

$=\dfrac{\cos 2\alpha(1+\sin 2\alpha)}{-\cos 2\alpha}=-(1+\sin 2\alpha)$;

4) $\dfrac{(\cos\alpha-\sin\alpha)^2}{\sin 2\alpha\cdot\cos 2\alpha-\cos 2\alpha}=\dfrac{1-2\cos\alpha\sin\alpha}{\cos 2\alpha(\sin 2\alpha-1)}=$

$=\dfrac{-(\sin 2\alpha-1)}{\cos 2\alpha(\sin 2\alpha-1)}=\dfrac{-1}{\cos 2\alpha}$.

354.

1) $\dfrac{\cos^2 x}{1-\sin x}-\sin(\pi-x)=\dfrac{\cos^2 x-\sin x(1-\sin x)}{(1-\sin x)}=\dfrac{1-\sin x}{1-\sin x}=1$;

2) $\dfrac{\cos^2 x}{1+\sin x}+\cos(1{,}5\pi+x)=\dfrac{\cos^2 x+\sin x(1+\sin x)}{1+\sin x}=\dfrac{1+\sin x}{1+\sin x}=1$;

3) $\dfrac{\sin^2 x}{1+\cos x}-\sin(1{,}5\pi+x)=\dfrac{\sin^2 x+\cos x(1-\cos x)}{1+\cos x}=\dfrac{1+\cos x}{1+\cos x}=1$;

4) $\dfrac{\sin^2 x}{1-\cos x}+\cos(3\pi-x)=\dfrac{\sin^2 x-\cos x(1-\cos x)}{1-\cos x}=\dfrac{1-\cos x}{1-\cos x}=1$.

355.

1) $tg\alpha+ctg\alpha=tg\alpha+\dfrac{1}{tg\alpha}=\dfrac{tg^2\alpha+1}{tg\alpha}=\dfrac{(\sin^2\alpha+\cos^2\alpha)\cos\alpha}{\cos^2\alpha\cdot\sin\alpha}=$

$=\dfrac{1}{\cos\alpha\sin\alpha}=\dfrac{1}{{}^1\!/_2\sin 2\alpha}=\dfrac{2}{\sin 2\alpha}$, т.к. $\alpha=-\dfrac{\pi}{12}$, то

$\sin 2\alpha=\sin\left(-\dfrac{\pi}{6}\right)=-\dfrac{1}{2}$ и значение выражения равно $\dfrac{2}{-{}^1\!/_2}=-4$;

2) $\text{ctg}\alpha-\text{tg}\alpha=\dfrac{\cos\alpha}{\sin\alpha}-\dfrac{\sin\alpha}{\cos\alpha}=\dfrac{\cos^2\alpha-\sin^2\alpha}{\cos\alpha\cdot\sin\alpha}=\dfrac{\cos 2\alpha}{{}^1\!/_2\sin\alpha}=2\text{ctg}2\alpha$.

Т.к. $\alpha=-\dfrac{\pi}{8}$, то $2\text{ctg}^2\alpha=2ctg\,(-\dfrac{\pi}{4})=2$;

3) $\dfrac{\cos\alpha}{\cos\alpha+\sin\alpha}+\dfrac{\sin\alpha}{\cos\alpha-\sin\alpha}=$

$$=\frac{\cos^2\alpha-\cos\alpha\cdot\sin\alpha+\cos\alpha\cdot\sin\alpha+\sin^2\alpha}{\cos^2\alpha-\sin^2\alpha}=\frac{1}{\cos 2\alpha}.$$

Т.к. $\alpha=-\dfrac{\pi}{6}$, то $\dfrac{1}{\cos 2\alpha}=\dfrac{1}{\cos\left(-\dfrac{\pi}{3}\right)}=\dfrac{1}{1/2}=2$;

4) $\dfrac{\sin\alpha}{\cos\alpha+\sin\alpha}-\dfrac{\cos\alpha}{\cos\alpha-\sin\alpha}=$

$$=\frac{\sin\alpha\cdot\cos\alpha-\sin^2\alpha-\cos^2\alpha-\cos\alpha\cdot\sin\alpha}{\cos^2\alpha-\sin^2\alpha}=\frac{-1}{\cos 2\alpha}.$$

Т.к. $\alpha=\dfrac{\pi}{3}$, то $\dfrac{1}{\cos 2\alpha}=\dfrac{-1}{\cos\dfrac{2\pi}{3}}=\dfrac{-1}{-1/2}=2$.

356.

$$\frac{2\sin\left(\dfrac{\pi}{2}+\alpha\right)\sin(\pi-\alpha)+\cos 2\alpha-1}{\cos 2\alpha+\sin\alpha\cdot\cos\alpha-\cos^2\alpha}=$$

$$=\frac{2\cos\alpha\sin\alpha+\cos^2\alpha-\sin^2\alpha-\cos^2\alpha-\sin^2\alpha}{\cos^2\alpha-\sin^2\alpha+\sin\alpha\cdot\cos\alpha-\cos^2\alpha}=\frac{2\sin\alpha(\cos\alpha-\sin\alpha)}{\sin\alpha(\cos\alpha-\sin\alpha)}=2$$

357.

1) $\sin(2x+3\pi)\sin\left(x+\dfrac{3\pi}{2}\right)-\sin 3x\cos 2x=-1$;

$-\sin 2x\cdot(-\cos 3x)-\sin 3x\cos 2x=-1$; $\sin(3x-2x)=1$, т.е. $\sin x=1$.

Тогда $x=\dfrac{\pi}{2}+2\pi n,\quad n\in Z$.

2) $\sin(5x-\dfrac{3\pi}{2})\cdot\cos(2x+4\pi)-\sin(5x+\pi)\sin 2x=0$;

$\cos 5x\cdot\cos 2x+\sin 5x\sin 2x=0;$
$\cos(5x-2x)=1;$
$\cos 3x=0.$

Тогда $3x=\dfrac{\pi}{2}+\pi n,\quad n\in Z$

и $x=\dfrac{\pi}{6}+\dfrac{\pi n}{3},\quad n\in Z.$

358.

1) $\text{tg}\,(\alpha+\beta)=\dfrac{\text{tg}\alpha+\text{tg}\beta}{1-\text{tg}\alpha\quad\text{tg}\beta}$, т.к. $\text{tg}\,\alpha=-\dfrac{3}{4}$, $\text{tg}\beta=2{,}4$, то

$$\text{tg}\,(\alpha+\beta)=\frac{3/4+2{,}4}{1+3/4\quad2{,}4}=\frac{1{,}65}{2{,}8}=\frac{165}{280}=\frac{33}{56};$$

2) $\text{ctg}\,(\alpha+\beta)=\dfrac{1}{\text{tg}(\alpha+\beta)}$. Т.к. $\text{ctg}\,\alpha=\dfrac{4}{3}$, то $\text{tg}\,\alpha=\dfrac{3}{4}$,

т.к. $\text{ctg}\,\beta=-1$, то $\text{tg}\,\beta=-1$; $\text{tg}\,(\alpha+\beta)=\dfrac{tg\alpha+tg\beta}{1-tg\alpha\cdot tg\beta}=$

$$=\frac{3/4-1}{1-3/4\cdot(-1)}=\frac{-1/4}{1\,3/4}=-\frac{1}{7},$$ поэтому $\text{ctg}\,(\alpha+\beta)=-7$.

359.

1) $$2\sin\left(\frac{\pi}{4}+2\alpha\right)\sin\left(\frac{\pi}{4}-2\alpha\right)=2\sin\left(\frac{\pi}{4}+2\alpha\right)\sin\left(\frac{\pi}{2}-\left(\frac{\pi}{4}+2\alpha\right)\right)=$$

$$=2\sin\left(\frac{\pi}{4}+2\alpha\right)\cos\left(\frac{\pi}{4}+2\alpha\right)=\sin\left(\frac{\pi}{2}+4\alpha\right)=\cos4\alpha;$$

2) $$2\cos\left(\frac{\pi}{4}+2\alpha\right)\cdot\cos\left(\frac{\pi}{4}-2\alpha\right)=2\cos\left(\frac{\pi}{4}+2\alpha\right)\cos\left(\frac{\pi}{2}-\left(\frac{\pi}{4}+2\alpha\right)\right)=$$

$$=2\sin\left(\frac{\pi}{4}+2\alpha\right)\cos\left(\frac{\pi}{4}+2\alpha\right)=\sin\left(\frac{\pi}{2}+4\alpha\right)=\cos4\alpha;$$

3) $$\cos^2(\frac{\pi}{4}-\alpha)-\cos^2(\frac{\pi}{4}+\alpha)=\cos^2(\frac{\pi}{2}-(\frac{\pi}{4}+\alpha))-\cos^2(\frac{\pi}{4}+\alpha)=$$

$$=\sin^2(\frac{\pi}{4}+\alpha)-\cos^2(\frac{\pi}{4}+\alpha)=-\cos^2(\frac{\pi}{2}+2\alpha)=\sin2\alpha;$$

4) $$\sin^2\left(\frac{\pi}{4}+\alpha\right)-\sin^2\left(\frac{\pi}{4}-\alpha\right)=\sin^2\left(\frac{\pi}{2}-\left(\frac{\pi}{4}-\alpha\right)\right)-\sin^2\left(\frac{\pi}{4}-\alpha\right)=$$

$$=\cos^2\left(\frac{\pi}{4}-\alpha\right)-\sin^2\left(\frac{\pi}{4}-\alpha\right)=\cos\left(\frac{\pi}{2}-2\alpha\right)=\sin2\alpha.$$

360.

1) $1+\cos2x=2\cos x$;
$2\cos^2x-2\cos x=0$;

2) $1-\cos2x=2\sin x$;
$2\sin^2x-2\sin x=0$;

$2\cos x\,(\cos x - 1) = 0$; $\begin{cases} \cos x = 0 \\ \cos = 1 \end{cases}$; $\begin{cases} x = \frac{\pi}{2} + \pi n,\ n \in Z \\ x = 2\pi k,\ k \in Z \end{cases}$;

$2\sin x\,(\sin x - 1) = 0$; $\begin{cases} \sin x = 0 \\ \sin x = 1 \end{cases}$; $\begin{cases} x = \pi n,\ n \in Z \\ x = \frac{\pi}{2} + 2\pi k,\ k \in Z \end{cases}$.

Глава V. Прогрессия

361.

1) $a_3 = 9$; $a_6 = 36$, $a_n = n^2$;

2) $a_k = 4$, если $k = 2$; $a_k = 25$, если $k = 5$;

$a_k = n^2$, если $k = n$; $a_k = (n + 1)^2$, если $k = n + 1$.

362.

1) Пусть $\underline{a_n = 2n + 3}$;

$a_1 = 2 \cdot 1 + 3 = 5$;

$a_2 = 2 \cdot 2 + 3 = 7$;

$a_3 = 2 \cdot 3 + 3 = 9$.

2) Пусть $\underline{a_n = 1 + 3n}$;

$a_1 = 1 + 3 \cdot 1 = 4$;

$a_2 = 1 + 3 \cdot 2 = 7$;

$a_3 = 1 + 3 \cdot 3 = 10$.

3) Пусть $\underline{a_n = 100 - 10n^2}$;

$a_1 = 100 - 10 \cdot 1 = 100 - 10 = 90$;

$a_2 = 100 - 10 \cdot 4 = 100 - 40 = 60$;

$a_3 = 100 - 10 \cdot 9 = 100 - 90 = 10$.

4) Пусть $\underline{a_n = \frac{n-2}{3}}$;

$a_1 = \frac{1-2}{3} = -\frac{1}{3}$;

$a_2 = \frac{2-2}{3} = 0$;

$a_3 = \frac{3-2}{3} = \frac{1}{3}$.

5) Пусть $\underline{a_n = \frac{1}{n}}$;

$a_1 = 1$; $a_2 = \frac{1}{2}$; $a_3 = \frac{1}{3}$.

6) Пусть $\underline{a_n = -n^3}$;

$a_1 = -1$; $a_2 = -8$; $a_3 = -27$.

363.

$x_n = n^2$

если $x_n = 100$, то $n = 10$; если $x_n = 144$, то $n = 12$

если $x_n = 225$, то $n = 15$

49, 169 – члены последовательности $x_n = n^2$, т.к. $49 = 7^2$, $169 = 13^2$

48 – не члены последовательности $x_n = n^2$.

364.

1) пусть $a_n = -3$, тогда $-3 = n^2 - 2n - 6$;

$n^2 - 2n - 3 = 0$. Решим: $n_1 = 3$; $n_2 = -1$ – не подходит, т.к. $n \in N$;

$\boxed{a_3 = -3}$ – член a_n;

2) пусть $a_n = 2$, тогда $2 = n^2 - 2n - 6$;

$n^2 - 2n - 8 = 0$. Решим: $n_1 = 4$; $n_2 = -2$ – не подходит, т.к. $n \in N$;

$\boxed{a_4 = 2}$ – член a_n;

3) пусть $a_n = 3$, тогда $3 = n^2 - 2n - 6$;

$n^2 - 2n - 9 = 0$. Решим: $\frac{D}{4} = 1 + 9 = 10$;

$n_{1,2} = \frac{1 \pm \sqrt{10}}{1}$ – не подходят, т.к. $n \in N$;

$a_n = n^2 - 2n - 6$ $a_n = -3$ – не член a_n;

4) пусть $a_n = 9$, тогда $9 = n^2 - 2n - 6$;

$n^2 - 2n - 15 = 0$. Решим: $n_1 = 5$; $n_2 = -3$ – не подходит, т.к. $n \in N$;

$\boxed{a_5 = 9}$ – член a_n.

365.

1) $a_2 = 3a_1 + 1 = 3 \cdot 2 + 1 = 6 + 1 = 7$;

$a_3 = 3a_2 + 1 = 3 \cdot 7 + 1 = 21 + 1 = 22$;

$a_4 = 3a_3 + 1 = 3 \cdot 22 + 1 = 66 + 1 = 67$;

2) $a_2 = 5 - 2a_1 = 5 - 2 \cdot 2 = 5 - 4 = 1$;

$a_3 = 5 - 2a_2 = 5 - 2 \cdot 1 = 5 - 2 = 3$;

$a_4 = 5 - 2a_3 = 5 - 2 \cdot 3 = 5 - 6 = -1$.

366.

1) Если $a_n = 150$, то

$150 = (n - 1)(n + 4)$;

$150 = n^2 + 3n - 4$;

$n^2 + 3n - 154 = 0$. Решим:

$D = 9 + 616 = 625 > 0$,

$n_{1,2} = \frac{-3 \pm 25}{2}$;

$n_1 = 11$, $\underline{n_2 = -14} \notin N$;

не подходит, т.к. $n \in N$.

Ответ: $n = 11$.

2) Если $a_n = 104$, то

$104 = (n - 1)(n + 4)$;

$104 = n^2 + 3n - 4$;

$n^2 + 3n - 108 = 0$. Решим:

$D = 9 + 432 = 441 > 0$,

$n_{1,2} = \frac{-3 \pm 21}{2}$;

$n_1 = 9$, $\underline{n_2 = -12} \notin N$;

не подходит, т.к. $n \in N$.

Ответ: $n = 9$.

367.

$a_2 = \sqrt{a_1} = \sqrt{256} = \sqrt{16^2} = 16$; $a_3 = \sqrt{a_2} = \sqrt{16} = \sqrt{4^2} = 4$;

$a_4 = \sqrt{a_3} = \sqrt{4} = \sqrt{2^2} = 2$.

368.

1) $a_2 = \sin\left(\frac{\pi}{2} \cdot a_1\right) = \sin\frac{\pi}{2} = 1$; $a_3 = \sin\left(\frac{\pi}{2} \cdot a_2\right) = \sin\frac{\pi}{2} = 1$;

$a_4 = \sin\left(\frac{\pi}{2} \cdot a_3\right) = \sin\frac{\pi}{2} = 1$; $a_5 = \sin\left(\frac{\pi}{2} \cdot a_4\right) = \sin\frac{\pi}{2} = 1$;

$a_6 = \sin\left(\frac{\pi}{2} \cdot a_5\right) = \sin\frac{\pi}{2} = 1$;

2) $a_2 = \cos\pi = -1$; $a_3 = \cos(-\pi) = -1$;

$a_4 = \cos\pi = -1$; $a_5 = \cos(-\pi) = -1$;

$a_6 = \cos\pi = -1$.

369.

$a_3 = a_1^2 - a_2 = 2^2 - 3 = 1$; $a_4 = a_2^2 - a_3 = 3^2 - 1 = 8$;

$a_5 = a_3^2 - a_4 = 1^2 - 8 = -7$.

370.

1) Пусть $a_n = -5_n + 4$;

$a_{n+1} = -5(n+1) + 4 = -5n - 5 + 4$;

$\boxed{a_{n+1} = -5n - 1}$;

$a_{n-1} = -5(n-1) + 4 = -5n + 5 + 4$;

$\boxed{a_{n-1} = -5n + 9}$;

$a_{n+5} = -5(n+5) + 4 = -5n - 25 + 4$;

$\boxed{a_{n+5} = -5n - 21}$.

2) Пусть $a_n = 2(n - 10)$.

Тогда $a_{n+1} = 2(n + 1 - 10) = 2n + 2 - 20$;

$a_{n+1} = 2n - 18$;

$a_{n-1} = 2(n - 1 - 10) = 2n - 2 - 20$;

$a_{n-1} = 2n - 22$;

$a_{n+5} = 2(n + 5 - 10) = 2n + 10 - 20$;

$a_{n+5} = 2n - 10$.

3) Пусть $a_n = 2 \cdot 3^{n+1}$. Тогда $a_{n+1} = 2 \cdot 3^{n+2}$;

$a_{n-1} = 2 \cdot 3^n$; $a_{n+5} = 2 \cdot 3^{n+6}$.

4) Пусть $a_n = 7 \cdot \left(\frac{1}{2}\right)^{n+2}$.

Тогда $a_{n+1} = 7 \cdot \left(\frac{1}{2}\right)^{n+3}$;

$a_{n-1} = 7 \cdot \left(\frac{1}{2}\right)^{n+1}$; $a_{n+5} = 7 \cdot \left(\frac{1}{2}\right)^{n+7}$.

372.

1) Т.к. $a_n = a_1 + (n-1)d$, то

$a_2 = 2 + 5 = 7;$

$a_3 = 7 + 5 = 12;$

$a_4 = 12 + 5 = 17;$

$a_5 = 17 + 5 = 22;$

2) Т.к. $a_2 = a_1 + d$, то

$a_2 = -3 + 2 = -1;$

$a_3 = -1 + 2 = 1;$

$a_4 = 1 + 2 = 3;$

$a_5 = 3 + 2 = 5.$

373.

1) $a_{n+1} = 3 - 4(n+1);$

$a_{n+1} - a_n = 3 - 4(n+1) - 3 + 4n = \cancel{3} - \underline{\underline{4\cancel{n}}} - 4 - \cancel{3} + \underline{\underline{4\cancel{n}}} = -4$,

т.к. разность $a_{n+1} - a_n$ не зависит от n, то это – арифметическая прогрессия.

2) $a_{n+1} = -5 + 2(n+1);$

$a_{n+1} - a_n = -5 + 2(n+1) + 5 - 2n = -\underline{\underline{\cancel{5}}} + \underline{2\cancel{n}} + 2 + \underline{\underline{\cancel{5}}} - \underline{2\cancel{n}} = 2$,

т.к. $a_{n+1} - a_n$ не зависит от n, то это – арифметическая прогрессия.

3) $a_{n+1} = 3(n+2);$

$a_{n+1} - a_n = 3(n+2) - 3(n+1) = \underline{\cancel{3n}} + 6 - \underline{\cancel{3n}} - 3 = 3,$

т.к. $a_{n+1} - a_n$ не зависит от n, то это – арифметическая прогрессия.

4) $a_{n+1} = 2(2-n);$

$a_{n+1} - a_n = 2(2-n) - 2(3-n) = 4 - \underline{2\cancel{n}} - 6 + \underline{2\cancel{n}} = -2$,

т.к. $a_{n+1} - a_n$ не зависит от n, то это – арифметическая прогрессия.

374.

1) $a_n = a_1 + (n-1)d$, $n = 15$, поэтому

$a_{15} = a_1 + 14d = 2 + 14 \cdot 3 = 2 + 42 = 44.$

Ответ: $a_{15} = 44$.

2) $a_n = a_1 + (n-1)d$, $n = 20$, тогда $a_{20} = a_1 + 19d;$

$a_{20} = 3 + 19 \cdot 4 = 3 + 76 = 79.$

Ответ: $a_{20} = 79$.

3) $a_n = a_1 + (n-1)d$, $n = 18$, тогда $a_{18} = a_1 + 17d;$

$a_{18} = -3 + 17 \cdot (-2) = -37.$

Ответ: $a_{18} = -37$.

4) $a_n = a_1 + (n-1)d$, $n = 11$, тогда $a_{11} = a_1 + 10d;$

$a_{11} = -2 + 10 \cdot (-4) = -42.$

Ответ: $a_{11} = -42$.

375.

1) $a_1 = 1$; $a_2 = 6$;

$d = 6 - 1 = 5;$

$a_n = a_1 + (n-1)d = 1 + (n-1) \cdot 5;$

$a_n = 5n - 4;$

2) $a_1 = 25$; $a_2 = 21$;

$d = 21 - 25 = -4;$

$a_n = a_1 + (n-1)d = 25 + (n-1) \cdot (-4);$

$a_n = -4n + 29;$

3) $a_1 = -4$; $a_2 = -6$;
$d = -6 - (-4) = -2$;
$a_n = a_1 + (n-1)d =$
$= -4 + (n-1) \cdot (-2)$;
$a_n = -2n - 2$;

4) $a_1 = 1$; $a_2 = -4$;
$d = -4 - 1 = -5$;
$a_n = a_1 + (n-1)d =$
$= 1 + (n-1) \cdot (-5)$;
$a_n = -5n + 6$.

376.

$a_1 = 44$; $d = 38 - 44 = -6$;
$a_n = a_1 + (n-1)d$. Тогда $-22 = 44 + (n-1) \cdot (-6)$;
$0 = 66 - 6n + 6$; $6n = 50 + 22$;
$6n = 72$; $n = 12$.

377.

$a_1 = -18$; $a_2 = -15$; $d = -15 - (-18) = 3$;
$a_n = a_1 + (n-1)d$.
Тогда $12 = -18 + (n-1) \cdot 3$;
$30 = 3n - 3$; $3n = 33$; $n = 11$.
Ответ: 12 является членом a_n.

378.

$a_1 = 1$; $a_2 = -5$; $d = -5 - 1 = -6$;
Тогда $-59 = 1 + (n-1) \cdot (-6)$;
$-60 = -6n + 6$;
$6n = 66$;
$n = 11$;
$a_{11} = -59$
является членом a_n.

$a_n = a_1 + (n-1)d$.
Значит $-46 = 1 + (n-1) \cdot (-6)$;
$0 = 47 - 6n + 6$;
$6n = 53$;
$n = 8\frac{5}{6}$ – не натуральное,
значит, -46 не является
членом a_n.

379.

1) $a_n = a_1 + (n-1)d$;
$a_{16} = a_1 + 15 \cdot d$, т.к. $a_1 = 7$, $a_{16} = 67$, то
$67 = 7 + 15d$; $15d = 60$. Отсюда $d = 4$.
2) $a_9 = a_1 + 8d$, т.к. $a_1 = -4$, $a_9 = 0$, то
$0 = -4 + 8d$; $8d = 4$. Тогда $d = \frac{1}{2}$.

380.

1) $\underline{a_9 = 12}$.
Т.к. $a_9 = a_1 + 8 \cdot d$, то
$12 = a_1 + 8 \cdot 1{,}5$;
$a_1 = 12 - 12$;
$a_1 = 0$.

2) $\underline{a_7 = -4}$.
Т.к. $a_7 = a_1 + 6 \cdot d$, то
$-4 = a_1 + 6 \cdot 1{,}5$;
$a_1 = -4 - 9$;
$a_1 = -13$.

381.

1) $\underline{d = -3; a_{11} = 20.}$

Т.к. $a_{11} = a_1 + 10d$, то

$20 = a_1 + 10 \cdot (-3)$;

$a_1 = 20 + 30 = 50$;

$a_1 = 50$;

2) $\underline{a_{21} = -10; a_{22} = -5,5}$;

$d = a_{22} - a_{21} = -5,5 - (-10) = 4,5$.

Т.к. $a_{21} = a_1 + 20 \cdot d$, то

$-10 = a_1 + 20 \cdot 4,5$;

$a_1 = -10 - 90 = -100$.

382.

1) <u>если $a_3 = 13; a_6 = 22$</u>.

Т.к. $a_6 = a_3 + 8d$, то

$22 = 13 + 3 \cdot d$.

Тогда $3d = 9$

и $d = 3$;

$a_3 = a_1 + 2d$;

$13 = a_1 + 2 \cdot 3$;

$a_1 = 13 - 6$.

Получим $a_1 = 7$.

Значит $a_n = a_1 + (n - 1)d$;

$a_n = 7 + (n - 1) \cdot 3$.

Итак, $a_n = 3n + 4$.

2) <u>если $a_2 = -7; a_7 = 18$</u>.

Т.к. $a_7 = a_2 + 5d$, то

$18 = -7 + 5d$.

Значит $5d = 25$

и $d = 5$;

$a_2 = a_1 + d$;

$a_1 = -7 - 5$.

Получим $a_1 = -12$.

Значит $a_n = a_1 + (n - 1)d$;

$a_n = -12 + (n - 1) \cdot 5$.

Итак, $a_n = 5n - 17$.

383.

$a_1 = 15; a_2 = 13$. Тогда $d = 13 - 15 = -2$.

Т.к. $a_n = a_1 + (n - 1)d$, то $a_n = 15 + (n - 1)(-2)$;

$a_n = -2n + 17$. Т.к. $a_n < 0$, то $-2n + 17 < 0$; $-2n < -17$.

Тогда $n > 8,5$, т.е. при $n \geq 9$ $a_n<0$.

384.

Т.к. $a_n = a_1 + (n - 1)d$, то $a_n = -10 + (n - 1)\cdot \frac{1}{2}$;

$a_n = \frac{1}{2}n - 10\frac{1}{2}$. Если $a_n < 2$, то $\frac{1}{2}n - 10\frac{1}{2} < 2$;

$n - 21 < 4$, $n<25$. Т.е. при $n \leq 25$; $a_n<2$.

385.

1) <u>если $a_8 = 126, a_{10} = 146$</u>;

$a_9 = \frac{a_8 + a_{10}}{2}$, тогда

$a_9 = \frac{126+146}{2} = \frac{272}{2} = 136$;

$d = a_9 - a_8$,

$d = 136 - 126 = 10$;

2) <u>если $a_8 = -64, a_{10} = -50$</u>;

$a_9 = \frac{a_8 + a_{10}}{2}$, тогда

$a_9 = \frac{-64-50}{2} = \frac{-114}{2} = -57$;

$d = a_9 - a_8$;

$d = -57 - (-64) = -57 + 64 = 7$;

3) $\underline{\text{если } a_8 = -7,\ a_{10} = 3}$;

$a_9 = \dfrac{a_8 + a_{10}}{2} = \dfrac{-7+3}{2} = \dfrac{-4}{2} = -2$;

$d = a_9 - a_8 = -2 - (-7) = 5$;

4) $\underline{\text{если } a_8 = 0{,}5,\ a_{10} = -2{,}5}$;

$a_9 = \dfrac{a_8 + a_{10}}{2} = \dfrac{0{,}5-2{,}5}{2} = -1$;

$d = a_9 - a_8 = -1 - 0{,}5 = -1{,}5$.

386.

Запишем данные условия: $a_5 = a_1 + 4d$.

Тогда $a_5 = 4{,}9 + 4 \cdot 9{,}8 = 44{,}1$ (м).

387.

Т.к. $a_n = a_1 + (n-1)d$,

то $105 = 15 + (n-1) \cdot 10$;

$90 = 10n - 10$;

$10n = 100$, отсюда $n = 10$.

Ответ: 10 дней.

388.

$a_n + a_k = a_1 + (n-1)d + a_1 + (k-1)d = 2a_1 + (n+k-2)d$,

но $a_{n-\ell} + a_{k+\ell} = a_1 + (n-\ell-1)d + a_1 + (k+\ell-1)d = 2a_1 + (n+k-2)d$,

тогда $a_n + a_k = a_{n-1} + a_{k+1}$, доказано,

поэтому $a_{10} + a_5 = a_{10-3} + a_{5-3} = a_7 + a_8 = 30$.

Ответ: $a_{10} + a_5 = 30$.

389.

$\dfrac{a_{n+k} + a_{n-k}}{2} = \dfrac{a_n + a_n}{2} = \dfrac{2a_n}{2} = a_n$ (из предыдущего номера),

тогда $a_{20} = \dfrac{a_{10} + a_{30}}{2} = \dfrac{120}{2} = 60$.

390.

1) $\underline{a_1 = 1,\ a_n = 20,\ n = 50;}$

$S_n = \dfrac{a_1 + a_n}{2} \cdot n$; $S_{50} = \dfrac{1+20}{2} \cdot 50 = (1+20) \cdot 25 = 21 \cdot 25 = 525$;

2) $\underline{a_1 = 1,\ a_n = 200,\ n = 100;}$

$S_{100} = \dfrac{1+200}{2} \cdot 100 = 201 \cdot 50 = 10050$;

3) $\underline{a_1 = -1,\ a_n = -40,\ n = 20;}$

$S_{20} = \dfrac{-1-40}{2} \cdot 20 = -41 \cdot 10 = -410$;

4) $\underline{a_1 = 2,\ a_n = 100,\ n = 50;}$

$S_{50} = \dfrac{2+100}{2} \cdot 50 = 102 \cdot 25 = 2550$.

391.

$a_n = 98$; $a_1 = 2$; $d = 1$. Т.к. $a_n = a_1 + (n-1)d$, то

$98 = 2 + (n-1)\cdot 1$;

$96 = n - 1$; $n = 97$;

$$S_{97} = \frac{2+98}{2}\cdot 97 = 50\cdot 97 = 4850.$$

392.

$a_1 = 1$; $d = 2$; $a_n = 133$.

Т.к. $a_n = a_1 + (n-1)d$, то

$133 = 1 + (n-1)\cdot 2$;

$132 = 2n - 2$; $n = 67$;

$$S_{67} = \frac{1+133}{2}\cdot 67 = 67\cdot 67 = 4489.$$

393.

1) $\underline{a_1 = -5;\ d = 0{,}5;}$

$$S_n = \frac{2a_1+(n-1)d}{2}\cdot n\ ;$$

$$S_{12} = \frac{2\cdot(-5)+11\cdot 0{,}5}{2}\cdot 12 =$$

$= (-10 + 5{,}5)\cdot 6 = -27$;

2) $\underline{a_1 = \frac{1}{2};\ d = -3;}$

$$S_n = \frac{2a_1+(n-1)d}{2}\cdot n\ ;$$

$$S_{12} = \frac{2\cdot\frac{1}{2}+11\cdot(-3)}{2}\cdot 12 =$$

$= (1 - 33)\cdot 6 = -192$.

394.

1) $a_1 = 9$; $d = a_2 - a_1 = 13 - 9 = 4$;

$$S_{11} = \frac{2a_1+10d}{2}\cdot 11 = \frac{2\cdot 9+10\cdot 4}{2}\cdot 11 = \frac{(18+40)\cdot 11}{2} = 29\cdot 11 = 319;$$

2) $a_1 = -16$; $d = a_2 - a_1 = -13 - (-16) = 6$ $S_{12} = \frac{2a_1+11d}{2}\cdot 12 =$

$$= \frac{2\cdot(-16)+11\cdot 6}{2}\cdot 12 = (-32+66)\cdot 6 = 6\cdot 34 = 204.$$

395.

1) $a_1 = 3$; $d = 3$; $a_n = 273$.

Т.к. $a_n = a_1 + (n-1)d$, то $273 = 3 + (n-1)\cdot 3$;

$270 = 3n - 3$; $3n = 273$.

Тогда $n = 91$.

$$S_{91} = \frac{a_1+a_{91}}{2}\cdot 91 = \frac{3+273}{2}\cdot 91 = 138\cdot 91 = 12558.$$

2) $a_1 = 90$; $d = 80 – 90 = – 10$; $a_n = – 60$.

Т.к. $a_n = a_1 + (n – 1)d$, то

$– 60 = 90 – 10n + 10$;

$10n = 100 + 60 = 160$.

Т.е. $n = 16$;

$$S_{16} = \frac{a_1 + a_{16}}{2} \cdot 16 = (90 – 60) \cdot 8 = 30 \cdot 8 = 240.$$